Calculus 2 - MTH 142 - Spring 2002 - NAME:

Practice for exam 1

SHOW YOUR WORK

In this first part, no calculators are allowed.

1 $\int \frac{6t+1}{t^2+2t^2+t} \, dt$  
2 $\int \frac{\ln(x^3)}{x} \, dx$  
3 $\int_{0}^{1} (-3x + 2)e^{2x} \, dx$  
4 $\int e^{\sin^2 x} \sin x \cos x \, dx$  
5 $\int \frac{2z}{2z-3} \, dz$  
6 $\int \frac{6t-4}{t^2+8t-9} \, dt$  
7 $\int \frac{-2}{x^2+x+1} \, dx$  
8 $\int \sin(e^x) \, e^x \, dx$  
9 $\int \ln(2t) \, dt$

Now calculators are allowed.

1. Use the midpoint rule with $n = 3$ to approximate

   $$\int_{0}^{3} \frac{1}{1+x} \, dx$$

   do the calculations by hand, write down all details.

2. The following table gives values of a function $f$, whose concavity does not change in the interval $[0, 2]$. We want to estimate $\int_{0}^{2} f(x) \, dx$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
<th>.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>1.492</td>
<td>2.08</td>
<td>2.48</td>
<td>2.75</td>
<td>2.92</td>
<td>3</td>
<td>2.98</td>
<td>2.86</td>
</tr>
</tbody>
</table>

   (a) Find the midpoint estimate with 4 subdivisions, Mid(4).

   (b) You can easily calculate that Left(4) = 3.915 and Right(4) = 5.345. Find the trapezoid and simpson’s estimates Trap(4) and Simp(4).

   (c) Is $f$ concave up or concave down. Justify your answer.

   (d) How many digits of accuracy would you say you have for sure with your answers for mid(4) and trap(4)? Consider that the actual value is between the two.

   (e) How much more accuracy would I get if I used n=40.

3. The numerical approximation of an integral $\int_{a}^{b} f(x) \, dx$ is 2.50. It is also known that left(5) = 2.532431 and mid(5) = 2.502215

   (a) What is the error in each case.

   (b) Give a reasonable guess for the error corresponding to left(50) and to mid(50).

   (c) How many decimals you predict will be correct when using mid(50) to calculate the integral?
4. Say which of the following integrals are improper. For those that are improper, determine if they are convergent or not.

A \( \int_{1}^{\infty} \frac{x}{1+x^6} \, dx \)  
B \( \int_{-2}^{5} \frac{x^2}{x+1} \, dx \)  
C \( \int_{0}^{5} \frac{2}{t^2+3t} \, dt \)  
D \( \int_{0}^{\infty} te^{-t} \, dt \)  
E \( \int_{1}^{\infty} \frac{z}{z^2+1} \, dz \)  
F \( \int_{0}^{1} t^2 \sin t \, dt \)

5. Determine if the improper integral is convergent, and give its value or prove that it is divergent.

(a) \( \int_{1}^{2} \frac{x^2}{x^2 - 1} \, dx \)

(b) \( \int_{1}^{2} \frac{x}{x^2 - 1} \, dx \)

(c) \( \int_{1}^{2} \frac{1}{x^2 - 1} \, dx \)

6. Integrate using methods of integration and formulas from the table of integrals. On the actual exam, the formulas will be given.

\( \int t \, \sin^4(t^2) \, dt \)

7. Integrate using trigonometric substitution.

\( \int \sqrt{3 + 4x^2} \, dx \)

8. Find the area of the region bounded by the y axis and the two functions

\( y = x^2, \ y = 6 - 3x \)

9. Find the volume of the solid given in your book, Figure 8.13, page 352. Do it either, way, by slicing into rectangular slices or triangular slices.

10. Suppose \( f'(x) = \sin(x^2) \) and \( f(2) = 1 \). Use the second fundamental theorem of calculus to write an expression for \( f(x) \).