1) \( f(x) = x^3 + x^2 - x - 1 \)

a) \( f'(x) = 3x^2 + 2x - 1 = 0 \)

\[ \Rightarrow (3x-1)(x+1) = 0 \]

\[ \Rightarrow x = \frac{1}{3} \text{ or } x = -1 \]

Thus \( \frac{1}{3}, -1 \) are critical numbers.

b) \( f''(x) = 6x + 2 \)

\[ f'' \left( \frac{1}{3} \right) = 4 > 0 \Rightarrow \text{CU} \Rightarrow \text{local min} \]

\[ f'' (-1) = -4 < 0 \Rightarrow \text{CD} \Rightarrow \text{local max} \]

c) \( f''(x) = 0 \) \( \Rightarrow \) \( x = -\frac{1}{3} \)

\[ f'' (-1) < 0 \Rightarrow x = -\frac{1}{3} \text{ is an inflection pt.} \]

\[ f'' (0) > 0 \]

de) no abs. min,
no abs. max.

f) \(-5 \leq x \leq 3\)

abs. min \(-9.6\)
abs. max \(3.2\)
2) a)  

B \[30 \text{ m/hr}\]

\[\frac{dy}{dt} = 40, \quad \frac{dx}{dt} = 30\]

find \(\frac{d^2}{dt^2}\) when \(z = \sqrt{(x - 15)^2 + (y - 25)^2}\)

so \(x^2 + y^2 = z^2\)

\[\Rightarrow \quad 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}\]

\[\Rightarrow \quad \frac{dz}{dt} = \frac{2(30)(15) - 2(40)(25)}{2 \sqrt{265}} \approx 49.5 \text{ m/hr}\]

c) Since Newton's method is essentially a root finder, we need to define a new function, \(f(x) = \ln x - 2\). Then we use Newton's method on that function. Note that,

\[x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\]
3) \[ \frac{2^y}{\ln 2} - \frac{1}{x} + C \]

4a) \[ \int_{-3}^{1} 3x^{\frac{1}{3}} \, dx = 3 \left( \frac{3}{5} x^{\frac{4}{3}} \right) \Bigg|_{-3}^{1} \]

= \[ 3 \left( \frac{3}{5} (1)^{\frac{4}{3}} - (\frac{3}{5} (-3)^{\frac{4}{3}}) \right) = 3 \left( -\frac{3}{5} - \frac{3}{5} \sqrt[3]{27} \right) \]

= \[ -\frac{9}{5} - \frac{9}{5} \sqrt[3]{27} \]

4b) \[ \int_{-3}^{1} \left( 2x^2 - \frac{2}{x} \right) \, dx = 2 \int_{-3}^{1} x^2 \, dx - 2 \int_{-3}^{1} \frac{1}{x} \, dx \]

= \[ 2 \left[ \frac{x^3}{3} \right]_{-3}^{1} - \ln |x| \Bigg|_{-3}^{1} \] = \[ 2 \left[ (-\frac{1}{3} + 9) - (\ln 1 - \ln 3) \right] \]

= \[ 2 \left[ \frac{26}{3} - - \ln 3 \right] = 2 \left( \frac{26}{3} + \ln 3 \right) \]

4c) \[ \int_{-3}^{1} \frac{y}{x} \, dx = 4 \int_{-3}^{1} \frac{1}{x} \, dx = 4 \ln |x| + C \]

4d) \[ \int_{-3}^{1} 2x^2 \, dx = -2 \int_{-3}^{1} 2x^2 \, dx = -2 \left( \frac{2x^3}{3} \right) + C \]

4e) \[ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = \sin x \Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \sin \frac{\pi}{2} - \sin 0 = 1 \]
(5) (a) \( \lim_{x \to \infty} \frac{x^2}{e^{2x}} = \lim_{x \to \infty} \frac{2x}{2e^{2x}} = 0 \)

(b) \( \lim_{x \to 0} x^2 \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x^2}} \)

\[ = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x \to 0} \frac{x^2}{2} = 0 \]

(c) \( \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = \lim_{\theta \to 0} \frac{-\sin \theta}{1} = 0 \)
(a) \[ \int_{0}^{3} \left( \frac{t^2}{2} - t \right) \, dt = 0 \]

(b) \[
\begin{align*}
\text{total distance} &= \left| \int_{0}^{2} \left( \frac{t^2}{2} - t \right) \, dt \right| + \left| \int_{2}^{3} \left( \frac{t^2}{2} - t \right) \, dt \right| \\
&= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}
\end{align*}
\]
\[ V = \pi R^2 H = 100 \text{ in}^3 \]

\[ H = \frac{100}{\pi R^2} \]

\[ C = 2 \cdot (\pi R^2 \cdot 2) + 2\pi R \cdot H = 4\pi R^2 + \frac{2\pi R \cdot 100}{1} \frac{1}{\pi R^2} \]

\[ C' = 8\pi R - \frac{200}{R^2} = 0 \]

\[ R \approx 2 \text{ is the minimum of } C \]
\[ \Delta x = \frac{2 - 1}{4} = \frac{1}{4} \]

\[ x_i = 1 + i \cdot \frac{1}{4} = 1 - \frac{i}{4} \]

\[ x_{i-1} = 1 - \frac{(i-1)}{4} \]

\[ f(x_i) = 2 \cdot \ln \left(1 + \frac{i}{4}\right) \]

\[ f(x_{i-1}) = 2 \cdot \ln \left(1 - \frac{(i-1)}{4}\right) \]

\[ L_y = \sum_{i=1}^{4} 2 \cdot \ln \left(1 + \frac{(i-1)}{4}\right) \cdot \frac{1}{4} \approx 0.594 \text{ (underestimate)} \]

\[ R_y = \sum_{i=1}^{4} 2 \cdot \ln \left(1 + \frac{i}{4}\right) \cdot \frac{1}{4} \approx 0.941 \text{ (overestimate)} \]

\[ L_y = (0.625) + (0.446)(0.25) + (0.811)(0.25) + (1.119)(0.25) \]

\[ R_y = (0.446)(0.25) + (0.811)(0.25) + (1.119)(0.25) + (1.386)(0.25) \]
9. \(\int_{0}^{10} 300 e^{0.03t} \, dt = 300 \left( \frac{1}{0.03} \right) \left[ e^{0.03 \cdot 10} - e^{0.03 \cdot 0} \right] \approx 1880 \)

\(1000 + \int_{0}^{10} 300 e^{0.03t} \, dt = 1000 + 1880 = 2880 \)

10. \(L_3 = (55)(5) + (60)(5) + (65)(5) = 900 \, \text{ft} \)

\(R_3 = 60 \cdot 5 + 65 \cdot 5 + 55 \cdot 5 = 900 \, \text{ft} \)

\(A_{ve} = 900 \, \text{ft} \)

11. a) area of \( \text{area} \) is 5

b) negative of the area of \( -(1 \cdot 2 \cdot \frac{1}{2} + \frac{1}{2} \cdot 2 \cdot 2) = -3 \)

c) \(5 - 3 = 2 \)