1. Let \( f(x) = \frac{1}{2x} \). The graph of \( f \) is shown below, for \( x \geq 0 \).

(a) On the graph above, sketch the tangent line at \((0.5, f(0.5))\).

(b) Find the exact slope of the line tangent to \( f(x) = \frac{1}{2x} \) at \( x = 0.5 \).

(c) Find the equation of the line tangent to \( f(x) = \frac{1}{2x} \) at \( x = 0.5 \).

2. The table gives the number of people, \( Q(t) \), infected by a virus \( t \) days since the outbreak of an epidemic in a small town.

<table>
<thead>
<tr>
<th>time (days)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number infected</td>
<td>22</td>
<td>60</td>
<td>120</td>
<td>350</td>
<td>670</td>
<td>785</td>
<td>810</td>
</tr>
</tbody>
</table>

(a) Estimate \( Q'(6) \) and explain its meaning. Don’t forget to include units in your answer.

(b) Suppose that \( Q'(20) = -40 \). Explain precisely what this means in terms of the epidemic.
3. For function $g(x)$ shown below

(a) For which values of $x$ on $[-3,3]$ is $g$ discontinuous?

(b) For which values of $x$ on $[-3,3]$ is $g$ not differentiable?

(c) By inspection of the graph, find $\lim_{h \to 0} \frac{g(1 + h) - g(1)}{h}$.
4. A ball is dropped from a tall building. Its height in feet at time $t$ seconds since being dropped is given by $H(t) = 256 - 16t^2$.

(a) Find the average velocity of the ball during the first two seconds.

(b) On the axes below, carefully sketch the graph of both $H$ and the secant line whose slope is the average velocity during the first two seconds.

(c) Find the velocity of the ball at time $t$.

(d) Use your answer from part (c) to find the velocity of the ball at time $t = 2$. On your graph, sketch the tangent line whose slope is the velocity at $t = 2$.

(e) How fast is the ball travelling when it hits the ground? (You must find when it hits the ground.)

(f) Find the acceleration of the ball at time $t$. 
5. The graph of a function $f$ is shown below.

(a) On which intervals of $[-2, 4]$ is $f$ increasing?
(b) On which intervals of $[-2, 4]$ is $f$ decreasing?
(c) On which intervals of $[-2, 4]$ is $f$ concave up?
(d) On which intervals of $[-2, 4]$ is $f$ concave down?
(e) For which $x$ in $[-2, 4]$ is the slope of the tangent of $f$ at $x$ equal to zero?
(f) On the same axes, carefully sketch the graph of the derivative of the function $f$.

6. Let $d(t) = 8(t^3 - 6t^2 + 12t)$ be the distance a car is from Tyler Hall in miles where $t$ is in hours. Is the car ever moving toward home? If so, when?

7. Let $f(t)$ be the number of centimeters of rainfall that has fallen since midnight, where $t$ is the time in hours.

(a) Describe the inverse function $f^{-1}$ in words. What are the domain and range of $f^{-1}$.

(b) Interpret the following in practical terms. Include units in your answers.
   i. $f(10) = 3.1$.
   ii. $f^{-1}(4.2) = 16$.
   iii. $f'(8) = 0.4$.
   iv. $(f^{-1})'(3.8) = 2.2$. 
8. Below is the graph of \( f(x) \). Place the following quantities in order, from lowest to highest:

\[
f'(a), \quad f'(b), \quad \frac{f(b) - f(a)}{b - a}, \quad \frac{f(c) - f(b)}{c - b}, \quad \frac{f(c) - f(a)}{c - a},
\]

where \( a = 0, \ b = 1, \) and \( c = 2 \). Hint: Represent each one as the slope of some line and compare slopes.

9. Let \( r(t) \) be the value of a stock at time \( t \) in months since the time of your purchasing it. The graph of \( r(t) \) is given below.

(a) Is \( r'(t) \) positive or negative?
(b) Is \( r''(t) \) positive or negative?
(c) What does \( r'(t) \) tell you about the value of the stock?
(d) What does \( r''(t) \) tell you about the value of the stock?
10. Let
\[ f(x) = \begin{cases} 
6x, & 0 \leq x \leq 2 \\
3x^2, & 2 \leq x 
\end{cases} \]
Graph \( f(x) \). Is \( f(x) \) continuous at \( x = 2 \)? Explain why it is not differentiable at \( x = 2 \).

11. Let \( g(x) = \ln x \). Find the equation of the tangent line at \( x = 1/e \) and use it to approximate \( g(0.3) \), rounded to 2 decimal places.

12. The figure below shows the graphs of a function, its first derivative, and its second derivative. Identify which is which.
13. The graph of a function $f$ is shown below. Sketch $f'$. 

14. Sketch the graph of a function that satisfies all of the given conditions.

- $f(-1) = 1$
- $f(1) = 0$
- $f'(-1) = f'(1) = 0$
- $f''(x) < 0$ if $x < 0$
- $f''(x) > 0$ if $x > 0$. 
15. The graph of $g'(x)$, the derivative of $g(x)$ is given.

![Graph of $g'(x)$]

(a) On which intervals of $[-1, 5]$ is $g$ increasing?
(b) On which intervals of $[-1, 5]$ is $g$ decreasing?
(c) On which intervals of $[-1, 5]$ is $g$ concave up?
(d) On which intervals of $[-1, 5]$ is $g$ concave down?
(e) For which $x$ in $[-1, 5]$ is the slope of the tangent of $g$ at $x$ equal to zero.
(f) Carefully sketch the graph of the antiderivative $g(x)$ of the function $g'(x)$, assuming $g(2) = 5$.

16. Let $C$ be the cost in dollars to build a square feet of a house. $C = f(a)$. Interpret the meaning of $f'(2000) = 100$.

Ans: The cost per square foot is $100 at the point when the number of square feet is 2000. In other words, to produce 2001 square feet would cost just $100 more than it does to produce 2000 square feet.
17. Let \( f(x) = 4x^3 + 6x^2 - 23x + 7 \). Find the intervals on which \( f(x) \) is increasing.

18. Let \( f(x) = x^4 - 4x^3 \). On what intervals is \( f(x) \) concave up?

19. Let \( f(x) = x^5 - 5x \). On what intervals is \( f(x) \) both increasing and concave up?

20. If \( f(x) = x^3 - 6x^2 - 15x + 20 \). Find analytically all the values of \( x \) for which \( f'(x) = 0 \). Show your answers on a rough sketch of the graph of \( f(x) \).

21. Find the following derivatives without using a calculator.

(a) \( y = 5t^2 + 4e^t \)
(b) \( y = 12e^x + 11x \)
(c) \( y = 3x - 2(4^x) \)
(d) \( f(x) = e^2 + x^e \)
(e) \( g(x) = \frac{1}{x^2} - \frac{1}{x^{3/2}} \)
(f) \( h(x) = x^5 - 3\sqrt{x} + 2 \)
(g) \( f(x) = (2x)^4 \)
(h) \( g(x) = (x^3)^5 \)
(i) \( y = xe^x \)
(j) \( F(x) = \sin^2 x \)
(k) \( y = \frac{e^x}{\sqrt{x}} \)
(l) \( j(x) = \frac{x^{4}-4x+3}{e^{x}+1} \)

22. Find the slope of \( f(x) = 1 - e^x \) at the point where it crosses the \( x \)-axis. Find the equation of the tangent line to the curve at this point.

23. Assume that \( f(x) \) and \( g(x) \) are differentiable functions about which we know very little. In fact, assume that all we know about these function sis the following table of data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3</td>
<td>1</td>
<td>-5</td>
<td>8</td>
</tr>
<tr>
<td>-1</td>
<td>-9</td>
<td>7</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>-3</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>3</td>
<td>8</td>
<td>?</td>
</tr>
</tbody>
</table>

(a) Let \( h(x) = e^x f(x) \). What is \( h'(0) \)?
(b) Let \( j(x) = -4f(x)g(x) \). What is \( j'(1) \)?
(c) Let \( k(x) = \frac{xf(x)}{g(x)} \). What is \( k'(-2) \)?
24. Find derivatives of the following functions without using a calculator.

(a) \( y = \frac{1 + \tan t}{1 - \tan t} \)
(b) \( y = \sin(2 - 3x) \)
(c) \( g(\theta) = \sin(2\theta) - \pi \theta \)
(d) \( y = \sin(e^t) \)
(e) \( y = \tan(x^2) \)
(f) \( y = \cos(\sin x) \)
(g) \( w = (t^3 + 1)^{100} \)
(h) \( w(r) = \sqrt{r^4 + 1} \)
(i) \( g(x) = e^{(2x + 7)} \)
(j) \( f(x) = e^{2x}(5^x) \)
(k) \( y = \ln(2x^2 + x) \)
(l) \( y = (\ln x)^2 \) same as \( \ln^2(x) \).
(m) \( y = \ln(\cos x) \)
(n) \( y = \log_2(2 + \sin x) \)

25. Find \( \frac{dy}{dx} \) implicitly where

\[ x + 2y^2 - xy = 6 \]

26. Use logarithmic differentiation to find the derivative of \( f(x) = x^{\sin x} \).

27. Use linear approximation of \( f(x) = \sin x \) at \( a = 0 \) to approximate \( \sin 1/3 \).