Show all work

1. The Board of Directors of the XYZ Corporation is holding an election to choose a new Chairman of the Board. The candidates are Allen, Beckman, Cole, Dent, and Emery. The preference schedule for the election is given in the following table.

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>10</th>
<th>8</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>A</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>Second choice</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Third choice</td>
<td>B</td>
<td>B</td>
<td>D</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>Fourth choice</td>
<td>D</td>
<td>E</td>
<td>A</td>
<td>E</td>
<td>B</td>
</tr>
<tr>
<td>Fifth choice</td>
<td>E</td>
<td>A</td>
<td>E</td>
<td>A</td>
<td>D</td>
</tr>
</tbody>
</table>

Find the winner of the election under the plurality-with-elimination method.

**Solution:**

Round 1: Eliminate E

Round 2: Eliminate C

Round 3: Eliminate D

Round 4: B has the majority, so B wins.

2. The members of the Tasmania State University soccer team are having an election to choose the captain of the team from among the four seniors- Anderson, Bergman, Chow, and Delgado. The preference schedule for the election is given in the following table.

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>4</th>
<th>1</th>
<th>9</th>
<th>8</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>Second choice</td>
<td>B</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>Third choice</td>
<td>D</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Fourth choice</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
</tbody>
</table>

Find the winner of the election using the Borda count method.

**Solution:**

A: 74
B: 51
C: 69
D: 76

D is the winner.
3. A math class is asked by the instructor to vote among four possible dates for the final exam - A (Dec. 15), B (Dec. 20), C (Dec. 21), and D (Dec. 23). The following is the class preference schedule.

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>10</th>
<th>3</th>
<th>8</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>D</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Second choice</td>
<td>B</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>A</td>
</tr>
<tr>
<td>Third choice</td>
<td>A</td>
<td>D</td>
<td>B</td>
<td>A</td>
<td>D</td>
</tr>
<tr>
<td>Fourth choice</td>
<td>C</td>
<td>B</td>
<td>A</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

Find the winner of the election using the pairwise comparisons method.

**Solution:**

A vs B: B wins
A vs C: A wins
A vs D: D wins
B vs C: B wins
B vs D: D wins
C vs D: D wins
D is the winner.

4. For this question, refer to the information given in problem number 3. This example shows that the plurality method fails to satisfy the condorcet criterion. Explain why.

**Solution:** D is a condorcet winner, as we see from the work shown in problem 3. Plurality gives C as the winner. Thus there is a condorcet winner who does not win the election. Using the plurality method to tally the votes.

5. Suppose there is an election between candidates A, B, C, D, E. The method of pairwise comparisons is used and the results of the pairwise comparisons are as follows.

A vs B: A wins
A vs C: A wins
A vs D: A wins
A vs E: E wins
B vs C: B wins
B vs D: B wins
B vs E: E wins
C vs D: D wins
C vs E: E wins
D vs E: E wins
Rank the candidates using the \textit{extended} pairwise comparisons method.

\textbf{Solution:} E,A,B,D, C

6. Candidates V, W, X, Y, and Z are running for election. In which of the situations below is the monotonicity criterion violated?

(a) The winner of the election is candidate Y. Candidate Z drops out. The ballots are recounted and now candidate X wins.

(b) Candidate X has the most first place votes, yet candidate Y wins the election.

(c) At first Y is the winner, but there is a reelection and some people change their preference lists, but each change only favors Y. Now in the tally of the votes in the 2nd election, X wins.

(d) None of the above.

\textbf{Solution:} The correct choice is (c).

7. Susan and Veronica want to divide an orange-pineapple cake worth $16 using the divider-chooser method. Susan puts a value of $14 on the orange side and $2 on the pineapple side while Veronica puts a value of $4 on the pineapple side and $12 on the orange side. Veronica is the divider of the cake.

(a) From the choices given, pick one cut that is a valid way for Veronica to cut the cake so that each piece is worth a fair share to Veronica.

The first cut gives 2/3 of the orange, vs the whole pineapple side plus 2/3 of the orange.

The second cut shows 5/6 of the orange side, vs the whole pineapple side plus 1/6 of the orange side.

\textbf{Solution:} Veronica values the first cut as two equal shares each worth $8.

(b) Using the cut you chose is part (a), show the value of each piece in Susan’s eyes?

\textbf{Solution:} Susan values the 2/3 of the Orange as $9.33 and the other piece as $6.67.

8. Three players (one divider and two choosers) are going to divide a plot of land fairly using the \textit{lone divider method}. Suppose the choosers value the sections as follows.

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chooser 1</td>
<td>35%</td>
<td>45%</td>
<td>20%</td>
</tr>
<tr>
<td>Chooser 2</td>
<td>30%</td>
<td>40%</td>
<td>30%</td>
</tr>
</tbody>
</table>

What is a fair division of the land?

\textbf{Solution:} Chooser 1 gets $s_1$. Chooser 2 gets $s_2$. The divider gets $s_3$.

9. In another situation with three players (one divider and two choosers), they are going to divide a plot of land fairly, again using the \textit{lone divider method}. Suppose the choosers value the sections as follows.
## 10.

Suppose David, Derek, and Chad are dividing a cake which is half vanilla and half strawberry fairly using the lone-chooser method. David and Derek are the dividers. Suppose in their eyes the values of the portions are as follows. David views the vanilla side as $6 and the strawberry side as $6. Derek views the vanilla side as $10 and the strawberry side as $2. Chad views the vanilla side as $4 and the strawberry side as $8.

Suppose that David makes the first cut as follows. Give the value that Derek places on each piece and show which one he selects. The cut gives 3/4 of vanilla and 1/4 strawberry with one piece and the other piece is 1/4 vanilla and 3/4 strawberry.

**Solution:** Derek values the piece with 3/4 of vanilla and 1/4 strawberry as $8.00 and the one with 1/4 vanilla and 3/4 strawberry as $4.00. So he chooses the first.

Now show how David cuts his piece and how Derek cuts his piece each into 3 fair shares.

**Solution:** Derek selected the piece with 3/4 of the vanilla side worth $7.50 to him and 1/4 of the strawberry side worth as $.50 to him. He divides it into 3 pieces each worth $2.67 by taking 2 completely in vanilla and the third is mixed, strawberry and vanilla.

David has the one with and the one with 1/4 of the vanilla worth $1.50 to him and 3/4 of the strawberry which he values as $4.50. He cuts it into 3 pieces each worth $2.00 with 2 all strawberry and one mixed.

Describe the resulting shares given to the 3 people.

**Solution:** It remains to see what Chad selected. Since he likes strawberry best, he selects the mixed piece from Derek and one of the all strawberry pieces from David. Derek ends up with the two all vanilla pieces. David ends up with one strawberry piece and on mixed piece.

## 11.

Robert and Peter equally inherit their parents’ old cabin and classic car. They decide to divide the 2 items using the method of sealed bids. Robert bids $29,200 on the car and $60,900 on the cabin. Peter bids $33,200 on the car and $55,300 on the cabin. Describe the final outcome of this fair-division problem.

**Solution:**
<table>
<thead>
<tr>
<th></th>
<th>Robert</th>
<th>Peter</th>
</tr>
</thead>
<tbody>
<tr>
<td>cabin</td>
<td>60,900</td>
<td>55,300</td>
</tr>
<tr>
<td>car</td>
<td>29,200</td>
<td>33,200</td>
</tr>
<tr>
<td>total</td>
<td>90,100</td>
<td>88,500</td>
</tr>
<tr>
<td>fair share</td>
<td>45,050</td>
<td>44,250</td>
</tr>
</tbody>
</table>

Robert gets the cabin and pays the estate $15,850.
Peter gets the car and $11,050 from the estate.
The estate has $4,800 left over and so pays each $2,400.

12. Four players (A, B, C, and D) agree to divide the 14 items shown below by lining them up in order and using the method of markers. The players’ bids are as indicated. Describe the initial allocation of the items to each player and name the pieces that are left over.

\[(1) (2)_{C_1} (3)_{A_1, B_1} (4)_{C_2} (5)_{A_2} (6)_{B_2} (7)_{D_1} (8)_{D_1} (9)_{A_3, D_3} (10)_{C_3} (11) (12)_{B_3} (13) (14)\]

**Solution:**

C: 1, 2
A: 4, 5
D: 9
B: 13, 14

Left over: 3, 6, 7, 8, 10, 11, 12.