6.3 (22) A steady wind blows a kite due west. The kite's height above the ground from horizontal position

\[ x = 0 \text{ to } x = 80 \text{ is } y = 150 - \frac{1}{50} (x - 50)^2 \]

Find the distance traveled by the kite.

\[ \frac{d}{dx} \left( \frac{1}{50} (x - 50)^2 \right)^\frac{1}{2} = \frac{x^2}{\sqrt{50}} \]

Sol: \[ \int_0^{80} \sqrt{1 + \left( \frac{d}{dx} \left( \frac{1}{50} (x - 50)^2 \right)^\frac{1}{2} \right)^2} \, dx \]

distance = \[ \int_0^{80} \sqrt{1 + \left( \frac{50}{x} \right)^2} \, dx \]

6.4 (12) The velocity graph of an accelerating car is shown.

(a) Estimate the average vel during first 12 sec.
(b) At what time was the instantaneous vel equal to the ave vel.

\[(\text{km/hr}) \]

\[ V \]

\[ \text{AV1} \]

\[ \text{t (sec)} \]

(a) Sol: Convert km/hr to km/sec.

\[ \frac{y \text{ km/hr}}{3600} = \frac{20 \text{ km}}{3600} \text{ km/sec} \]

\[ \text{Vel ave} = \frac{\int_0^{12} \text{Vel dx}}{12} \approx \frac{1}{12} (27) \text{ km/sec} \]

\[ 1 \text{ box} = 2 \text{ sec} \cdot 10 \text{ km/hr} = 20 \text{ km} \]

\[ = \frac{1}{12} (27) \left( \frac{20}{3600} \right) \text{ km/sec} = \frac{45}{3600} \text{ km/hr} \]

(b) Approx 5 sec
Q6 - continued

A471 43

Find the slope of the tangent line to
\[ r = 1 + \cos \theta \]
at the point where \( \theta = \frac{\pi}{6} \). Sketch.

So,
\[
\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}
\]

\[
\frac{dr}{d\theta} = -\sin \theta
\]

\[
\begin{align*}
\sin \frac{\pi}{6} &= \frac{1}{2} \\
\cos \frac{\pi}{6} &= \frac{\sqrt{3}}{2}
\end{align*}
\]

\[
\left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{6}} = \frac{-\sin \theta \sin \theta + (1 + \cos \theta) \cos \theta}{-\sin \theta \cos \theta - (1 + \cos \theta) \sin \theta}
\]

\[
\left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{6}} = \frac{-\left( \frac{1}{2} \right) \cdot \frac{1}{2} + (1 + \frac{\sqrt{3}}{2}) \frac{\sqrt{3}}{2}}{-\frac{1}{2} \cdot \frac{\sqrt{3}}{2} - (1 + \frac{\sqrt{3}}{2}) \cdot \frac{1}{2}}
\]

\[
= -1
\]

\[
\frac{dr}{d\theta} = -\sin \theta
\]

\[
\begin{align*}
\sin \frac{\pi}{6} &= \frac{1}{2} \\
\cos \frac{\pi}{6} &= \frac{\sqrt{3}}{2}
\end{align*}
\]