MTH 142 Summer 2005 - QUIZ 3 (On 5.8 and 5.9)

Name:

Write neatly and show all work.

1. The following formula is taken from the Table of Indefinite Integrals in our book.

\[ \int \tan x \, dx = - \ln |\cos x| + C \]

Derive the formula by rewriting \( \tan x = \frac{\sin x}{\cos x} \) and using a suitable substitution.

\[
\int \frac{\sin x}{\cos x} \, dx = \int \frac{1}{\cos x} \, \sin x \, dx
\]

\[
= \int \frac{1}{u} \, du = - \ln |u| = - \ln |\cos x| + C
\]

2. The following formulas are taken from the Table of Indefinite Integrals in our book.

\[ \int \frac{1}{\sin^m x} \, dx = \frac{-1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m > 0 \]

\[ \int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C \]

Use them to solve the following integral.

\[ \int \frac{1}{\sin^3 x} \, dx = \frac{-1}{2} \frac{\cos x}{\sin^2 x} + \frac{1}{2} \int \frac{1}{\sin x} \, dx \]

\[ \frac{1}{2} \frac{\cos x}{\sin^2 x} + \frac{1}{4} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C \]
3. Use (a) Trap(4) and (b) Mid(4) to estimate \( \int_0^1 \sin(x^2) \, dx \) using. From a graph of the integrand, decide whether your answers are underestimates or overestimates.

Using Table on calculator

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x_0 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>.125</td>
<td>.25</td>
<td>.375</td>
<td>.5</td>
<td>.625</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>.01562</td>
<td>.06246</td>
<td>.14016</td>
<td>.2474</td>
<td>.38077</td>
</tr>
</tbody>
</table>

\[
L(4) = \frac{1}{4} \left[ 0 + .06246 + .2474 + .5333 \right] = .21079
\]

\[
R(4) = \frac{1}{4} \left[ .06246 + .2474 + .5333 + .84147 \right] = .421158
\]

\[
\text{Trap}(4) = \frac{L + R}{2} = \frac{.21079 + .421158}{2} = .315974
\]

\[
\text{Mid}(4) = \frac{1}{4} \left[ .01562 + .14016 + .38077 + .69299 \right]
\]

\[
= .307385
\]