Show all work

A group of twelve students have to decide among three types of pizza: Sausage(S), Mushroom (M), and Beef(B). Their votes are given in the following preference schedule.

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>First choice</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>B</td>
<td>M</td>
</tr>
<tr>
<td>Second choice</td>
<td>M</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Third choice</td>
<td>S</td>
<td>S</td>
<td>B</td>
<td>M</td>
<td>B</td>
</tr>
</tbody>
</table>

1. Which choice will the group make if they use plurality voting?

2. Which choice will the group make if they use Borda Count?

3. Which choice will the group make if they use Plurality with elimination?

4. Is there a Condorcet Winner?

5. What does your answer to (d) say, if anything, about the answers to (a), (b), and (c) above?

Solutions:

1. 
   \[ B : 5, \quad M : 4, \quad S : 3 \]
   B, Beef, is the winner

2. 
   \[ B : 9 + 6 + 3 + 6 + 1 = 25 \]
   \[ M : 6 + 9 + 6 + 2 + 3 = 26 \]
   \[ S : 3 + 3 + 9 + 4 + 2 = 21 \]
   M, Mushroom, is the winner

3. Round 1: Eliminate S. Round 2: B:5, M:7, So M has the majority.
   M, Mushroom is the winner.

   So M, Mushroom is the Condorcet Winner.