Exam 3 covers Sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.8, 4.9, 5.1, 5.2, and 5.3

1. Let \( f(x) = x^3 + x^2 - x - 1 \)

(a) Find critical numbers.

(b) Test each critical number to determine if it is an local maximum, local minimum, or neither.

(c) Find inflection points by first finding \( x \)-values for which \( f''(x) = 0 \) and then testing each one to determine if it is an inflection point.

(d) Carefully, sketch a graph of the function with critical points and inflection points plotted exactly.

(e) What is the absolute maximum and absolute minimum of this function?

(f) What is the absolute maximum and absolute minimum when we restrict the value of \( x \) to \(-5 \leq x \leq 3\)?

2. (15 pts.) Two cars approach the same intersection on two streets which are perpendicular: Car A travels south at 40 mph, and Car B travels east at 30 mph.

(a) Draw a diagram to represent the situation.

(b) Suppose that at exactly 12:00, Car A is 0.45 miles north of the intersection while Car B is 0.25 west of the intersection. Assume that the speeds remain constant. How fast is the distance between the cars changing at 12:00?

(c) We wish to find the value of \( x \) for which \( \ln x = 2 \). Describe how you would use Newton’s Method to do so. Provide the formula that expresses \( x_{i+1} \) as a function of \( x_i \).

3. Provide the family of antiderivatives for \( g(x) = 2x + \frac{1}{x^2} \).

4. Evaluate the definite or indefinite integral.

(a) \( \int_{-3}^{-1} 3x^{2/3} \, dx \)

(b) \( \int_{-3}^{-1} 2x^2 - \frac{2}{x} \, dx \)

(c) \( \int \frac{4}{x} \, dx \)

(d) \( \int -4e^{2x} \, dx \)
5. Find the limit
   (a) \( \lim_{x \to \infty} \frac{x^2}{e^{2x}} \)
   (b) \( \lim_{x \to 0} x^2 \ln x \)
   (c) \( \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} \)

6. The velocity of a particle along a straight line at \( t \) seconds is \( v(t) = \frac{t^2}{2} - t \) ft/sec. Find
   (a) The change in position (the displacement) of the particle from \( t = 0 \) to \( t = 3 \) seconds.
   (b) The total distance travelled on \( 0 < t < 3 \).

7. A cylindrical can with a volume of 100 in\(^3\) is to be constructed. The material used for the bottom and top costs 2 times as much as the side material. Find the dimensions of the can that minimizes the cost of material.

8. Find \( L_4 \) and \( R_4 \) for the function \( 2 \ln x \) on the interval \([1,2]\). Determine whether each value an overestimate or underestimate. Show all steps. Check your answer with the calculator program \textit{Left Right Sums}.

9. A population of bacteria grows at the rate of \( 300e^{0.03t} \) organisms per minute, with \( t \) in minutes.
   (a) Set up a definite integral that expresses the number of organisms by which the population increased in the time \( t = 5 \) to \( t = 10 \) minutes?
   (b) Suppose that 10000 bacteria are present at time \( t = 5 \) How many are present 5 minutes later?

10. The speed of a car along a straight road at various times is given in the table below. Find the best estimate possible of the distance travelled from \( t = 25 \) seconds to \( t = 40 \) seconds. (Hint: You should use the average of \( L_3 \) and \( R_3 \).)

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed (ft/sec)</td>
<td>20</td>
<td>40</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
<td>55</td>
<td>50</td>
<td>55</td>
</tr>
</tbody>
</table>

11. The graph of a function \( f(x) \) is shown in the figure. Let \( F(x) = \int_{-2}^{x} f(t) \, dt \). Find
   (a) \( \int_{-3}^{0} f(x) \, dx \) exactly
(b) \( \int_{0}^{3} f(x) \, dx \) exactly

(c) \( \int_{-3}^{3} f(x) \, dx \) exactly

Figure 1: \( f(x) \)