1. Approximate answers are acceptable for this problem.

(a) \( f (\text{noon}) = 87^\circ, f (\text{6 P.M.}) = 67^\circ, \) range of \( f \) is \([53, 87] \).

(b) \( f \) is increasing on \((6, 12)\) and \((20, 22)\); \( f \) is decreasing on \((0, 6), (12, 20)\) and \((22, 24)\).

(c) Possible explanations for the drop in temperature at noon are a sudden thundershower, or an air conditioner being turned on.

(d) A possible explanation for \( f \) attaining its minimum value at 6 A.M. is that this is just before sunrise.

2. The total volume is the volume of a cylinder of height and radius \( r \) plus the volume of a hemisphere of radius \( r \), that is, \( V = \pi r^2 h + \frac{2}{3} \pi r^3 \).

3. (a) If we buy 8 cards for $2.80, then this costs less than buying 6 individual cards at $0.50 apiece. Hence, \( C(8) = 2.80 \).

(b) \[
\begin{align*}
c(x) &= \begin{cases} 
25 + 0.5(x - 80) & \text{if } 80 \leq x \leq 85 \\
27.8 & \text{if } 86 \leq x \leq 88 \\
27.8 + 0.5(x - 88) & \text{if } 89 \leq x \leq 90 
\end{cases}
\end{align*}
\]

(d) To buy 1005 cards, the best deal is to buy one carton (800 cards), two boxes (160 cards), five packs (40 cards) and five individual cards. The total cost is \( 230 + 2(25) + 5(2.80) + 5(0.50) = 296.50 \).

4. (a) \[
\begin{align*}
\end{align*}
\]

(b) \[
\begin{align*}
\end{align*}
\]
5. (a) \((f \circ g)(2) = f(0) = 1\)

(b) \((g \circ f)(2) = g(3) = 1\)

(c) \((f \circ f)(2) = f(3) = 4\)

(d) \((g \circ g)(2) = g(0) = 4\)

(e) \((f + g)(2) = f(2) + g(2) = 3 + 0 = 3\)

(f) \(\left(\frac{f}{g}\right)(2)\) is undefined because \(g(2) = 0\).

(g) \(g^{-1}(2)\) is undefined because \(g(x)\) takes on the value 2 twice, for \(x = 0.6\) and \(x = 3.4\).

6. \(f(x) = \begin{cases} 
-x - 1 & \text{if } x < -1 \\
\sqrt{1 - x^2} & \text{if } -1 \leq x \leq 1 \\
-x + 1 & \text{if } x > 1 
\end{cases}\)

7. (a) \(g(x) = 3x^2 + 3x\)

(b) \(g(4) = 60, f(4) = 59.023837\)

(c) The percentage error in using \(g(4)\) as an approximation for \(f(4)\) is \(100 \left| \frac{f(4) - g(4)}{f(4)} \right| = 1.65\%\).

(d) For larger values of \(x\), \(g(x)\) is an overestimate of \(f(x)\) because the coefficient of the dominant term \((x^2)\) is larger.

8. \(f^{-1}(x) = x^5 + 2x^3 + 3x + 1\)

(a) \(f^{-1}(1) = 7, f(1) = 0\)

(b) The value \(x_0\) such that \(f(x_0) = 1\) is \(f^{-1}(1) = 7\).

(c) The value \(y_0\) such that \(f^{-1}(y_0) = 1\) is \(f(1) = 0\).
(d) The graph of $f(x)$ is the graph of $f^{-1}(x)$ reflected about the line $y = x$.

9. Let $f(x) = 3 \cdot 2^{-x} + 1$. Then $f(x)$ is always decreasing, has a horizontal asymptote at $y = 1$, and $f(0) = 4$.

10. (a) $f(50,000) - f(49,999)$ represents the cost of producing the 50,000th disc.
    (b) $f^{-1}(10)$ represents the number of discs that can be made for $10,000$.
    (c) The cost per disc is cheapest for $30,000 < a < 40,000$. This is where the slope of $f$ is the smallest.
    (d) One possible explanation for the sudden increase in the curve’s slope is scarcity of materials.

11. (a) $x_1(t) = \frac{1}{2}x(t) = \frac{1}{2}(t \sin t), y_1(t) = y(t) = t + \cos t$
    (b) $x_2(t) = -x(t) = -t \sin t, y_2(t) = \frac{1}{2}y(t) = \frac{1}{2}(t + \cos t)$

12. (a) ![Graph](image1)
    (b) ![Graph](image2)
    (c) $\lim_{x \to \infty} \ln \ln x = \infty$

13. (a) Use $f(1) = 1$ and $f(3) = 0$ to get $A = -\frac{1}{\ln 3}$ and $B = 1$.
    (b) Use $f(0.2) = 0$ to get $K = 5$. 

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1. Describe what Figure 1.5 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

As the number of workers grow, the productivity also grows but the rate at which it grows slows down, possibly due to workers getting in each other's way.

2. Right after a certain drug is administered to a patient with a rapid heart rate, the heart rate plunges dramatically and then slowly rises again as the drug wears off. Sketch a possible graph of the heart rate against time from the moment the drug is administered.

3. Generally, the more fertilizer that is used, the better the yield of the crop. However, if too much fertilizer is applied, the crops become poisoned, and the yield goes down rapidly. Sketch a possible showing the yield of the crop as a function of the amount of fertilizer applied.

4. Graph the following pairs of functions in the same window on your graphing calculator. Draw a picture of what you see for each pair, labeling axes and indicating the scale. Zoom in enough to notice differences. Note that $\log_a(x) = \frac{\log_{10}(x)}{\log_{10}(a)}$. 
(a) $\log_{10} x, \log_2 x$

(b) $\log_2 (x^3), \log_2^3 x$

(c) $3^x, 2^x$

(d) $x^2, 2^x$

$x^2 = 2^x$ when $x = 2$ and $x = 4.$
5. Mary starts at a point A at time \( t = 0 \) and walks in a straight line at a constant rate of 6 mi/hr toward a point B which is 45 miles away. Express the time \( t \) as a function of her distance \( s \) from B.

The function is linear.

\[
\begin{array}{c|c}
\text{Time} & \text{Distance} \\
\hline
0 & 0 \\
7.5 & 90 \\
\end{array}
\]

\[
\frac{\Delta t}{\Delta s} = \frac{7.5}{90} = 0.0833 \\
The t-intercept is 0. So, \( t = 1.67s \).
\]

6. Express the cosine function as a horizontal shift of the sine function. That is, find \( h \) such that \( \cos x = \sin(x + h) \).

\[
\cos x \text{ is the sine function shifted to the left by } \frac{\pi}{2} \text{ units. So, } \\
\cos x = \sin \left(x + \frac{\pi}{2}\right)
\]

7. Are all polynomials rational functions? Write a sentence, and explain your answer. Give an example of a rational function which is not a polynomial.

Yes, all polynomials are rational. The rational function is any polynomial divided by a polynomial. So given a polynomial, divide it by the special polynomial 1. The rational function \( \frac{1}{x} \) is not a polyn.
8. Let \( g(t) = 2t^2 - t \). Find an expression in terms of \( \Delta t \) for the following:

\[
\frac{g(1 + \Delta t) - g(1)}{\Delta t}
\]

Reduce your answer if possible.

\[
\frac{2(1 + \Delta t)^2 - (1 + \Delta t) - [2 - 1]}{\Delta t} = \frac{2 + 4\Delta t + 2(\Delta t)^2 - 1 - \Delta t - 1}{\Delta t}
\]

\[
= \frac{3(\Delta t)^2 + 3\Delta t}{\Delta t} = 2\Delta t + 3
\]

9. Sketch the graphs of \( \cos\left(\frac{x}{2}\right) \) and \( \frac{1}{2}\cos(x) \). Make sure to label the axes appropriately. You may use your graphing calculator.

10. Sketch the graph of the function \( f \), where

\[
f(x) = \begin{cases} 
3x - x^2 & x \geq 3 \\
x - 3 & 1 < x < 3 \\
x^2 + 4x - 7 & x \leq 1 
\end{cases}
\]

(3, 6) 
(1, -2)
11. Sketch the graph of the function \( f(x) \), where
\[
f(x) = \begin{cases} 
3x - x^2 & x \geq 3 \\
x - 3 & 1 < x < 3 \\
x^2 + 4x - 6 & x \leq 1 
\end{cases}
\]

12. The following is the graph of the function \( y = g(x) \). Sketch \( y = 2g(x) \), \( y = 2 - g(x) \) and \( y = 1/g(x) \). To do so, in each case, make a table of values for \( y \) when \( x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( 2g(x) )</th>
<th>( 2-g(x) )</th>
<th>( 1/g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>DNE</td>
</tr>
<tr>
<td>-4</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>.5</td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>.5</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>.5</td>
</tr>
<tr>
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<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>DNE</td>
</tr>
<tr>
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<td>-2</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>-4</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>-4</td>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

13. Suppose the function \( f(t) = 5^t \) represents the number of new people receiving a chain letter at step \( t \) in the chain. How many steps does it take before the number of new people reaches 1 billion?

Solve for \( t \): 
\[
5^t \geq 1,000,000,000 = 10^9
\]

\[
\log_{10} 5^t = \log_{10} 10^9
\]

\[
t \log_{10} 5 = 9
\]

\[
t = \frac{9}{\log_{10} 5} = 12.87
\]

Step 12: \( 5^{12} \leq 10^9 \)
Step 13: \( 5^{13} \) is more.
14. Simplify:

\[
\begin{align*}
\log_{10}(10^5) &= 5 \\
\log_{10}(10^{-5}) &= -5 \\
\log_{10} 1 &= 0
\end{align*}
\]

15. Give a table for \(\log_{10}(x)\) for the values \(x = 1/4, 1/3, 1/2, 1, 2, 3, 4\). Graph it and label the axes carefully. What is the domain of this function?

<table>
<thead>
<tr>
<th>(x)</th>
<th>(\log_{10} x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/4)</td>
<td>(-0.6021)</td>
</tr>
<tr>
<td>(1/3)</td>
<td>(-0.4771)</td>
</tr>
<tr>
<td>(1/2)</td>
<td>(-0.301)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.30103</td>
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<td>3</td>
<td>0.47712</td>
</tr>
<tr>
<td>4</td>
<td>0.60206</td>
</tr>
</tbody>
</table>

Domain: \(\forall x \in x \geq 0\)
Sample Exam Solutions

1. (a) \( \lim_{t \to 0^+} f(t) = \infty \), \( \lim_{t \to 0^-} f(t) = 1 \), \( \lim_{t \to 2^-} f(t) = 3 \), \( \lim_{t \to \infty} f(t) = 1 \)
   
   (b) \( \lim_{t \to 2} f(t) \) exists for all \( x \) except \( x = 0 \) and \( x = 2 \).
   
   (c) There is a vertical asymptote at \( x = 0 \).
   
   (d) There is a horizontal asymptote at \( y = 1 \).
   
   (e) \( f \) is discontinuous at \( x = 0, 2, \) and \( 4 \).

2. Solve \( 3(2) + 1 = 2a + b \) and \( 5^2 = 5a + b \) to get \( a = 6, b = -5 \).

3. Taking \( \lim_{x \to \infty} f(x) \) gives a horizontal asymptote at \( y = a \). Algebraic simplification gives a vertical asymptote at \( x = -a \). The function is undefined at \( x = 0 \), but there is no asymptote there because \( \lim_{x \to 0} f(x) = 0 \).

4. \( f(x) = \frac{x + 4}{(x + 4)(x - 1)} \)
   
   (a) The domain is all values of \( x \) except \( x = 1 \) and \( x = -4 \).
   
   (b) Algebraic simplification gives a limit of \(-\frac{1}{5}\).
   
   (c) \( f \) is not continuous at \( x = -4 \), for it is not defined there. It can be modified by defining \( f(-4) \) to be \(-\frac{1}{5}\).

5. (a) C. True for \( f(x) = x \), untrue for \( f(x) = x^2 + x - 1 \)
   
   (b) A. True by the Intermediate Value Theorem
   
   (c) C. True for \( f(x) = x \), untrue for \( f(x) = x^2 + x - 1 \)
   
   (d) A. True by the Intermediate Value Theorem
   
   (e) C. True for \( f(x) = x \), untrue for \( f(x) = x^2 + x - 1 \)
   
   (f) B. \( \lim_{x \to 0} f(x) \) does not exist, contradicting the continuity of \( f \).

6. (a) Answers will vary. Look for:
   
   (i) zeros at 1 and 2
   
   (ii) \( f' \) positive for \( x \in (0, 1) \) and \( (2, 4) \)
   
   (iii) \( f' \) negative for \( x \in (1, 2) \)
   
   (iv) \( f' \) flattens out for \( x > 2.5 \)

   (b) Answers will vary. Look for
   
   (i) \( F(0) = 1 \)
   
   (ii) \( F \) is always increasing
   
   (iii) \( F \) is never perfectly flat
   
   (iv) \( F \) is closest to being flat at \( x = 2 \)
   
   (v) \( F \) is concave up for \( x \in (0, 1) \) and \( x \in (2, 4) \)
   
   (vi) \( F \) is concave down for \( x \in (1, 2) \)
7. (a) \[
\frac{3-(\frac{-1}{2})}{2-4} = \frac{2}{3}
\]
(b) The equation of the tangent line is \[y - 3 = -\frac{2}{3}(x + 2), \] so \[f(3) = -\frac{2}{3}(3 + 2) + 3 = -\frac{1}{3}.\]
(c) The equation of the tangent line is \[y - 3 = -\frac{2}{3}(x + 2).\]

8. Answers will vary; the following are samples only.
(a) \(f(x) = x^2, g(x) = x\)
(b) \(f(x) = 6x, g(x) = x\)
(c) \(f(x) = x, g(x) = x^2\)
(d) This is not possible. For \(\lim_{x \to \infty} \frac{f(x)}{g(x)} = -1\), either \(f\) or \(g\) would have to be negative for large \(x\). This contradicts the assumption that \(\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = \infty\).

9. Answers will vary.
(a) \(f(x) = x^2, a = 3\)
(b) \(f(x) = 2^x, a = 1\)
(c) \(f(x) = (x + 1)^{3/2}, a = 3\)
(d) \(f(x) = \sin(\pi x), a = 2\)

10. (a) \(\sqrt{2}\) (ii) \(1\) (iii) Does not exist (iv) \(9\) (v) \(9\) (vi) \(9\) (vii) \(3\) (viii) \(3\)
(b) \(f\) is discontinuous at \(x = 1\).

11. \(f\) isn't differentiable at \(x = 1\), because it is not continuous there; at \(x = -2\), because it has a vertical tangent there; and at \(x = 4\), because it has a cusp there.

12. (a) Answers will vary. One good answer would be to compute the average speed between 1 and 2 (14 ft/s) and the average speed between 2 and 3 (18 ft/s) and average them to get 16 ft/s. This is also the answer obtained by computing the average speed between 1 and 3.
(b) Answers will vary. Using reasoning similar to the previous part, we get an estimate of 22 ft/s, but it could be argued that a number closer to 22.5 would be more accurate.
(c) Answers will vary. The average speed between \(t = 5\) and \(t = 6\) is 22.5 ft/s.
(d) Since we are given information only about the cyclist's position at one-second intervals, we cannot determine if the speed is constantly increasing.

13. (a) \(\frac{1}{2}\) (b) \(0\) (c) Does not exist, because \(\lim_{x \to -1} f(x) = 0\) while \(\lim_{x \to -1} g(x) = 1\).
(d) \(-4\) (e) \(1\) (f) \(2\) (g) \(0\)

14. (a) \(f\) is increasing on \((-1, 1)\).
(b) Local minimum at \(x = -1\); local maximum at \(x = 1\)
(c) \(f\) is concave up where \(f''(x)\) is increasing, that is, on \((-2, 0)\).