1. Solve the IVP and check your answer:

\[(1-x^2)y'' - xy' + y = 0\]
\[y(0) = 0, \ y'(0) = 1\]

Ans: Solution

What does the existence and uniqueness theorem predict for this IVP?

Why?

Find a recurrence formula for the terms of the power series.
2. (a) What does the existence and uniqueness theorem predict for the IVP:

\[ xy'' + y' = x \]
\[ y(-1) = 0, \ y'(-1) = 1 \]?

Justify your answer.

(b) Solve the above IVP:
3. (a) Solve by Laplace Transforms, the IVP:
\[
\begin{align*}
\dot{y} + 5y + 6y &= 4 \\
y(0) &= -3, \quad \dot{y}(0) = 1
\end{align*}
\]

(b) Compute, by using the definition, the \( \mathcal{L}[-1 + e^{-2t}] \)
NAME:

4. Solve the IVP, by the method of elimination:

\[ \begin{align*}
\dot{x} &= -x - y \\
\dot{y} &= x + y
\end{align*} \]

and check your answer.

\( x(0) = 1, \ y(0) = -1 \)
5 (a) State the existence and uniqueness theorem for nonlinear DEs.

(b) What does the above theorem predict for the IVP

\[
\begin{align*}
   y' &= -2x^2 + 1 + y^2 \\
   y(0) &= 0
\end{align*}
\]

(c) Use Euler's Method with \( h = 0.2 \) to solve the above IVP in the interval \( 0 \leq x \leq 1 \).
6. In an RLC-series circuit: $R = 6$ ohms, $C = \frac{1}{4}$ farads, $L = 2$ henry and $V(t) = 6 e^t$ volts. Assuming that initially $Q(0) = 0$ and $I(0) = 0$, find the current in the circuit at any time $t$. Graph your answer. Find the charge in the capacitor after a very long time.

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**Answers:**

**Current:**

**Charge in the capacitor after a very long time:**

**Graph of Current:**

$\text{Graph of Current:}$
7. (a) Solve the BVP \[ y'' + y = 1 + A \]
where A is a real number:
\[ y(0) = y'(\pi) = 2 \]

(b) For what values of A does there exist a unique solution?
What is the solution?

(c) For what values of A do there exist infinitely many solutions?
What are the solutions?

(d) For what values of A do there exist no solutions?
Why?
8. What does the existence and uniqueness theorem predict for each of the following three IVPs? Why?

Solve the IVP.

(a) \( y' = y^{1/3}, y(0) = 0 \)

(b) \( y' = y^3, y(0) = 0 \)

(c) \( (1 + x^2) y'' - xy = 0 \)

\[ \begin{align*}
    y(0) &= y'(0) = 0
\end{align*} \]

Solutions:

(a) Sol.

(b) Sol.

(c) Sol.
9. A spring is stretched 2 cm by a force of 8 dynes. A mass of 2 g is attached to the end of the spring. The system is then set into motion by pulling the mass 6 cm above the point of equilibrium and releasing it, at time \( t = 0 \), with an initial velocity of 6 cm/sec. Assume also that air resistance acts upon the mass. This air resistance force equals six times the velocity of the mass at time \( t \).

Find the position of the mass as a function of time. Investigate the behavior of the motion as \( t \to \infty \).

Position of mass:

Behavior of motion as \( t \to \infty \):
10. Find a particular solution of the DE:

\[ y'' - 2y' + y = -3e^x + 2e^x \]