Lesson 11 - Network Flows

$v, w$-disconnecting set

Edge form of Menger’s Theorem: The maximum number of edge-disjoint paths connecting two distinct vertices $u$ and $w$ of a connected graph is equal to the minimum number of edges in a $v, w$-disconnecting set.

$v, w$-separating set

Menger’s Theorem: The maximum number of vertex-disjoint paths connecting two distinct non-adjacent vertices $v$ and $w$ of a graph is equal to the minimum number of vertices in a $v, w$-separating set.

Network $N$ - weighted digraph, with source vertex $v$ (indegree=0) and sink vertex $w$ (outdeg=0), for arc $a$, $\psi(a)$ is called the capacity of $a$ and is a function from the arc set to the positive reals. A flow is an assignment to each arc $a$ a non-negative real number $\phi(a)$ so that $\phi(a) \leq \psi(a)$ and so that for all vertices $x$ not equal to $v$ or $w$, $\sum$ flow into $x = \sum$ flow out of $x$.

Value of flow = flow out of $v$ = flow into $w$.

Max Flow Min Cut theorem: In any network, the value of any maximum flow is equal to the capacity of any minimum cut.

Applications: distribution, traffic vehicles per hour, electrical networks.