1. Write a truth table to show that $\sim (P \Rightarrow Q)$ is equivalent to $P \land \sim Q$.
2. Let $E$ be the set of even numbers. Let $P$ be the statement: $\exists M \in \mathbb{R}, \forall n \in E, |n| \leq M$.
   Form the negation of $P$ and determine whether or not $P$ is true. Give your reason.
3. Write a truth table to show that $P \Rightarrow Q$ is equivalent to $\sim Q \Rightarrow \sim P$.
4. (a) For $i \in \{0, 1, 2, 3, 4, 5\}$, let $N_i = \{n \in \mathbb{Z} : n$ is a multiple of $i\}$. The collection, $\mathcal{M} = \{N_i : 0 \leq i \leq 5\}$, is a family of subsets of $\mathbb{Z}$. Is $\mathcal{M}$ a partition of $\mathbb{Z}$? Explain your answer.
   (b) For $i \in \{0, 1, 2, 3, 4, 5\}$, let $N_i = \{n \in \mathbb{Z} : n$ divided by $6$ has a remainder of $i\}$. The collection, $\mathcal{M} = \{N_i : i \in \{0, 1, 2, 3, 4, 5\}\}$, is a family of subsets of $\mathbb{Z}$. Is $\mathcal{M}$ a partition of $\mathbb{Z}$? Explain your answer.
   (c) For $i \in \{0, 1, 2\}$, let $D_i = \{n \in \mathbb{Z} : n$ divided by $3$ has a remainder of $i\}$. The collection, $\mathcal{D} = \{D_i : i \in \{0, 1, 2\}\}$, is a family of subsets of $\mathbb{Z}$. Is $\mathcal{D}$ a partition of $\mathbb{Z}$? Explain your answer.
5. (a) Name all of the 2-element subsets of $\{0, 1, 2, 3\}$.
   (b) Name all of the subsets of $\{1, 2, 3\}$.
   (c) How many subsets in $\mathcal{P}(n)$, the power set of the set $[n] = \{1, 2, 3, \ldots, n\}$?
6. Show that $(A \cap B) \times C \subseteq (A \times C) \cap (B \times C)$.
7. You go to a deli for lunch and choose a drink, a sandwich, and a pie. The choices for drinks are coffee, soda, or water. The choices for a sandwich are turkey or vegie, and the choices for pie are apple, blueberry, or pumpkin. Draw a tree diagram that illustrates all of your options. How many options do you have?
8. Prove that $A \cap (B \cap C) \subseteq (A \cap B) \cap (A \cup C)$ directly, without using any theorems.
9. Use a truth table to show $((\sim P) \land (\sim Q)) \lor Q$ is equivalent to $P \Rightarrow Q$.
10. Let $\mathcal{O}$ be the set of odd integers. Rewrite the following statement using symbols: $\forall, \exists, \mathcal{O}$, and $\mathbb{Z}$. Also, establish its truth. Include a proof. Hint: look up what we gave in class as the definition of odd numbers.
    Every odd integer can be written in the form $2k - 1$ for some integer $k$.
11. Establish the truth of the following statement (include a proof) and negate it.
    $\forall a, b \in \mathbb{R}, a \neq b$ implies $a^2 - b^2 \neq 7$.
12. Establish the truth of the following statement (include a proof) and negate it.
    $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x \leq n$.
13. Establish the truth of the following statement (include a proof) and negate it.
    $\exists n \in \mathbb{N}, \forall x \in \mathbb{R}, x \leq n$.
14. Consider the statement about real numbers $a$ and $x$.
    $\forall x \in \mathbb{R}$, if $ax = 0$ then $x = 0$.
    (a) Form the contrapositive.
    (b) Form the converse.
    (c) Form the negation.
    (d) Is (a) true or false? Explain.
    (e) Is (b) true or false? Explain.
(15) Rewrite the following statement using quantifiers, establish its truth, and then negate it.

Every real number is less than 100.

(16) Rewrite the following statement using quantifiers, establish its truth, and then negate it.

Every even number greater than 2 is the product of an even number and a prime number.

For $k \in \{1, 2, \ldots, 10\}$, let $D_k = \{n \in \mathbb{N} | n \text{ divides } k \text{ evenly} \}$. So that $D_{10} = \{1, 2, 5, 10\}$.

(a) Find $D_1$, $D_2$, and $D_6$.
(b) Find $|D_1|$, $|D_2|$, and $|D_6|$.
(c) Find $\mathcal{P}(D_{10})$.

(17) Let $S = \{n \in \mathbb{Z} | n \text{ is even} \}$ and $T = \{n \in \mathbb{Z} | n \text{ is evenly divisible by } 3\}$.

(a) Name 3 different elements of $S \cap T$.
(b) Name 3 different elements of $S \times T$.

(18) Let $\mathcal{C} = \{(n, \infty) | n \in \mathbb{Z}\}$. Prove that $\mathcal{C}$ is nested. Be sure to look at the definition of nested on page 12 of the text.