1) Proof Th 32.4 (c): Assume that \( f'(c) < 0 \) for all interior points of \( I \). To show that \( f \) is strictly decreasing in \( I \), take \( x, y \in I \), \( y > x \). \( f \) satisfies the assumptions of the NVT in \( [x, y] \). Hence, there exists \( c \in (x, y) \) such that
\[
f'(c) = \frac{f(y) - f(x)}{y - x}
\]
Since \( f'(c) < 0 \), \( y - x > 0 \), we have \( f(y) - f(x) < 0 \).
Thus \( f(y) < f(x) \). Since \( x, y \in I \) were arbitrary, we have proved that \( f \) is strictly decreasing.

(d) can be proved similarly.

3) Let \( h = f - g \). Then \( h \) is continuous on \( I \) and for each interior point \( x \) of \( I \) we have
\[
h'(x) = f'(x) - g'(x) = 0.
\]
Thus, by Th. 32.3, \( h \) is constant in \( I \); that is, for some constant \( c \):
\[
h(x) = f(x) - g(x) = c \quad \text{for all} \quad x \in I.
\]
The latter implies
\[
f(x) = g(x) + c \quad \text{for all} \quad x \in I.
\]
2) The converse of (a) is not true. Take, for example, 
\( f(x) = x^3 \). \( f \) is strictly increasing in \( I = (-\infty, +\infty) \), 
yet \( f'(0) = 0 \).

The converse of (b) is:

"If \( f \) is increasing in \( I \), then \( f'(x) \geq 0 \) for all 
interior points \( x \) of \( I \)."

The latter implication is true. Assume \( f \) is increasing 
in \( I \). Let \( x \) be an interior point of \( I \). Since \( f \) is 
differentiable at \( x \) we have

\[
f'(x) = \lim_{y \to x} \frac{f(y) - f(x)}{y - x} = \lim_{y \to x^+} \frac{f(y) - f(x)}{y - x}.
\]

Since \( f \) is increasing

\[
\frac{f(y) - f(x)}{y - x} \geq 0 \quad \text{whenever} \quad y \in I, \quad y > x.
\]

Thus

\[
f'(x) = \lim_{y \to x^+} \frac{f(y) - f(x)}{y - x} \geq 0.
\]

4) For the sake of contradiction suppose that \( f \) 
has two fixed points \( d_1, d_2 \in \mathbb{R} \). Without any loss 
of generality, we may assume \( d_1 < d_2 \). Notice that 
\( f \) satisfies the assumptions of the MVT in \( \overline{d_1, d_2} \).
Thus, there exists $c \in (d_1, d_2)$ such that
\[ f'(c) = \frac{f(d_2) - f(d_1)}{d_2 - d_1} . \]

Since $d_1, d_2$ are fixed points $f(d_1) = d_1, f(d_2) = d_2$. Therefore
\[ f'(c) = \frac{d_2 - d_1}{d_2 - d_1} = 1. \]

Contradiction as $f'(c) < 1$. We conclude that $f$ has at most one fixed point.

5) (a) Observe that $p(0) = 1$, $p(-1) = -2$. The IVT gives that there exists $c \in (-1, 0)$ such that $p(c) = 0$.

(b) As $p(x) > 0$ for all $x > 0$, all roots of $p(x)$ must be negative. Let $r_1, r_2$ be two distinct negative roots of $p$, $r_1 < r_2$. From Rolle's Theorem, there exists $d \in (r_1, r_2)$ such that $p'(d) = 0$. (Indeed, $p(r_1) = p(r_2) = 0$.) But $p'(x) = 7x^6 + 5x^4 + 3x^2 > 0$ for all $x < 0$. Contradiction. Therefore, $p$ has exactly one real root.