1) You prove Th 26. 4 by minor modifications of the proof of Th. 19. 1.

2) (a) Let $c \in \mathbb{R}$. Let $f$ be defined in some interval $(c, a)$. We say that

$$\lim_{x \to c^+} f(x) = +\infty$$

if

$$\forall \varepsilon > 0 \exists \delta > 0 \forall x \in (c, a) (c < x < c + \delta \Rightarrow f(x) > M).$$

(b) Let $c \in \mathbb{R}$. Let $f$ be defined in a deleted nbhd $N_d$ of $c$. We say that

$$\lim_{x \to c} f(x) = -\infty$$

if

$$\forall \varepsilon > 0 \exists \delta > 0 \forall x \in N_d (0 < |x - c| < \delta \Rightarrow f(x) < -M).$$

(c) Let $f$ be defined in some interval $(a, +\infty)$. Let $L \in \mathbb{R}$. We say that

$$\lim_{x \to +\infty} f(x) = L$$

if

$$\forall \varepsilon > 0 \exists K \in \mathbb{R} \forall x \in (a, +\infty) (x > K \Rightarrow |f(x) - L| < \varepsilon).$$

(d) Let $f$ be defined in some interval $(a, +\infty)$. We say

$$\lim_{x \to +\infty} f(x) = +\infty$$

if

$$\forall M > 0 \exists K \in \mathbb{R} \forall x \in (a, +\infty) (x > K \Rightarrow f(x) > M).$$
3) Let's first define limit at a point being $+\infty$.

**Def.** Let $c \in \mathbb{R}$, $f$ be defined in a deleted nbhd $N_d$ of $c$. We say that

$$
\lim_{x \to c} f(x) = +\infty
$$

if

$$
\forall M > 0 \quad \exists \delta > 0 \quad \forall x \in N_d, \quad |x - c| < \delta \implies f(x) > M. \tag{1}
$$

Consider $f(x) = \frac{1}{(x-1)^2}$, $c = 1$. We shall prove

$$
\lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty. \tag{2}
$$

To prove (2), take an arbitrary $M > 0$. Let $\delta = \sqrt{\frac{1}{M}}$. Let $x \in N_d$ be such that

$$
0 < |x - 1| < \delta.
$$

Then

$$
\frac{1}{(x-1)^2} = \frac{1}{|x-1|^2} > \frac{1}{\delta^2} = \frac{1}{1/M} = M.
$$

Since $M$ was arbitrary, (1) holds and (2) is proved.

4) Let $x_0 \in (0, 1)$ be arbitrary. We shall prove $f(x_0) = 0$. From the density of rationals, for every $n=1,2,\ldots$ we can choose $q_n \in (0,1) \cap \mathbb{Q}$ such that

$$
q_n \in (x_0 - \frac{1}{n}, x_0 + \frac{1}{n}). \tag{3}
$$

(3) implies that $\lim_{n \to \infty} q_n = x_0$. Since $q_n \in \mathbb{Q}$ for $n=1,2,\ldots$ we have

$$
f(q_n) = 0 \quad \text{for } n=1,2,\ldots \tag{4}
$$

From Th.19.1, $\lim_{n \to +\infty} f(q_n) = f(x_0)$. But $\lim_{n \to +\infty} f(q_n) = 0$ by (4).

Therefore, $f(x_0) = 0$. 

5) By Prop. 21.2 and continuity of $f$ and $g$ in $\mathbb{R}$ we have
\[ \lim_{x \to a} f(x) = f(a) \quad \text{and} \quad \lim_{x \to a} g(x) = g(a). \quad (5) \]
Take $\varepsilon = \frac{f(a) - g(a)}{2}$. Since $f(a) > g(a)$, $\varepsilon > 0$. By (5) there exist $\varepsilon_1, \varepsilon_2$ both positive and such that
\[ |f(x) - f(a)| < \varepsilon \text{ whenever } |x - a| < \varepsilon_1, \quad (6) \]
\[ |g(x) - g(a)| < \varepsilon \text{ whenever } |x - a| < \varepsilon_2. \quad (7) \]
Take $\delta = \min \{ \varepsilon_1, \varepsilon_2 \}$. Then $\delta > 0$ and by (6) and (7)
\[ |f(x) - f(a)| < \varepsilon \text{ and } |g(x) - g(a)| < \varepsilon \text{ whenever } |x - a| < \delta, \]
In other words, for all $x \in (a - \delta, a + \delta)$ we have
\[ f(a) - \varepsilon < f(x) < f(a) + \varepsilon, \quad g(a) - \varepsilon < g(x) < g(a) + \varepsilon. \quad (8) \]
By the choice of $\varepsilon$ we have $f(a) - \varepsilon = g(a) + \varepsilon$. Hence, by (8) we obtain
\[ f(x) > g(x) \text{ for all } x \in (a - \delta, a + \delta). \]
6) Straightforward from the definition of the limit of a sequence.