5) Since $A \neq \emptyset$, there exists $a \in A$. Hence:
\[ \inf A \leq a \leq \sup A. \]
Thus:
\[ \inf A \leq \sup A. \]
Let $A = \{a\}$ be a set with one element. From the definition of the supremum and the infimum, we obtain easily:
\[ \inf A = a = \sup A. \]

6) (a) We have to prove for a fixed $a \in \mathbb{R}$, $a \neq -1$:
\[ 1 + a + a^2 + \ldots + a^n = \frac{1-a^{n+1}}{1-a} \quad \text{for all } n \in \mathbb{N}. \quad (1) \]

We use FIP.

(i) For $n=1$ the formula becomes:
\[ 1 + a = \frac{1-a^2}{1-a}. \]
Since $1-a^2 = (1-a)(1+a)$, the formula is true.

(ii) Let $k \in \mathbb{N}$ be given. Assume:
\[ (IA) \quad 1 + a + \ldots + a^k = \frac{1-a^{k+1}}{1-a}. \]

We have to prove that
\[ 1 + a + \ldots + a^k + a^{k+1} = \frac{1-a^{k+2}}{1-a}, \quad (2). \]

Indeed, by (IA):
\[ 1 + a + \ldots + a^k + a^{k+1} = (1 + a + \ldots + a^k) + a^{k+1} = \]
\[ = \frac{1-a^{k+1}}{1-a} + a^{k+1} = \frac{1-a^{k+1}+a^{k+1}}{1-a} = \frac{1-a^{k+2}}{1-a}. \]

Thus (2) holds, and, by FIP, (1) holds.
(b) Let $a \in \mathbb{R}$, $a > -1$. We have to prove that
\[(1 + a)^n \geq 1 + na \quad \text{for all } n \in \mathbb{N}. \quad (3)\]
We use FIP.

(i) For $n = 1$ the formula (3) becomes
\[1 + a \geq 1 + a\]
and is true.

(ii) Let $k \in \mathbb{N}$ be given. Assume that
\[(1 + a)^k \geq 1 + ka \quad (4).\]
We have to prove that
\[(1 + a)^{k+1} \geq 1 + (k+1)a. \quad (5)\]

Indeed, by (EA):
\[(1 + a)^{k+1} = (1 + a)^k (1 + a) \geq (1 + ka) (1 + a) = 1 + ka + a + ka^2 = 1 + (k+1)a + ka^2 \geq 1 + (k+1)a.\]
The latter inequality holds as $ka^2 \geq 0$. Thus (5) holds, and, by FIP, (3) holds.

7) For every $ka \in B$ we have $ka \leq ks$ as $k > 0$ and $a \leq s$ for all $a \in A$. Thus $ks$ is an upper bound for $B$. Suppose $p < ks$ is an upper bound for $B$. Then, $ka \leq p < ks$ for all $a \in A$. Thus, for all $a \in A$:
\[a \leq \frac{1}{k} p < s,\]
Contradiction as $s = \sup A$. If $k \leq 0$, $ks = \inf B$. Prove it yourself.