Exam 2 is scheduled for Friday, Nov 21, 3-5, Lippitt 204. Exam covers classes 17-29, homework assignments 5-8.

Exam 2 covers material that is fundamental in mathematical analysis including continuity, uniform continuity, differentiability, the Max-Min Theorem, the Intermediate Value Theorem and the Mean Value Theorem. Therefore, you have to study all the material thoroughly.

You are expected to know and be able to state all definitions, theorems and propositions given in class. You are expected to know all proofs done in class. You should be familiar with all examples presented in class. During classes 17-29 we studied some very important examples. You should know solutions to all homework problems. On the exam, you will have to state some definitions, theorems and propositions given in class as well as prove things. Some proofs will come from those given in class. Some exam problems will be similar to homework problems. In addition, some problems not done in class and not similar to homework problems will appear. Below are a few questions of the kind you should expect.

- State the definition of the limit of a function \( f \) at a point \( c \). (Def 17.4).
- State Heine’s characterization of the limit of a function \( f \) at a point \( c \). (Def 18.2).
- Prove that the limit \( \lim_{x \to 0} \sin \frac{1}{x} \) does not exist. (Ex. 19.1)
- Prove that the limit \( \lim_{x \to 0} x \sin \frac{1}{x} \) does exist. (Ex. 17.1)
- Show that the Dirichlet function \( D(x) \) has no limit at any point \( c \).
- Prove (a) or (b) or (c) or (d) of Th.19.1.
- State the definition of a continuous function \( f \) on an interval \( I \). (Def 20.1).
- State Heine’s characterization of continuity of a function \( f \) on an interval \( I \). (Def 21.2).
- State the definition of a bounded function \( f \) on an interval \( I \).
- State the Boundedness Theorem. (Th. 22.2)
- Prove the Boundedness Theorem. (Th. 22.2)
- State the Min-Max Theorem. (Th. 23.1)
- Prove the Min-Max Theorem. (Th. 23.1)
- State the Intermediate Value Theorem. (Th. 23.2)
- Prove the Intermediate Value Theorem. (Th. 23.2)
- Prove that the range of a function \( f \) that is continuous and not constant on a closed bounded interval \([a, b]\) is a closed bounded interval. (Prop. 24.1)
- State the definition of a uniformly continuous function \( f \) on an interval \( I \).
- Prove that \( f(x) = \frac{1}{x} \) is not uniformly continuous on \((0, 1]\). (Ex. 25.2)
- Prove that \( f(x) = \frac{1}{x} \) is uniformly continuous in \([1, +\infty)\).
- State the definition of differentiability of a function \( f \) at a point \( x_0 \). (Def. 26.1)
- Show that if a function \( f \) is differentiable at \( x_0 \), then \( f \) is continuous at \( x_0 \). (Th.26.1)
- Give an example of a function that is differentiable at 0 but not continuous at any \( x \neq 0 \). (Ex. 26.3)
- Prove (a) or (b) or (c) or (d) of Th. 27.1
- Prove that if $f$ is differentiable at $x_0$ and has a local extremum at $x_0$, then $f'(x_0) = 0$. (Th. 28.2)
- State Rolle’s Theorem. (Th. 28.3)
- Prove Rolle’s Theorem. (Th. 28.3)
- State the Mean Value Theorem. (Th. 29.1)
- Prove the Mean Value Theorem. (Th. 29.1)
- Prove Th. 29.2.
- Prove any part of Th. 29.3.

Homework problems that strike me as particularly good candidates to possibly include in the
exam are: Homework 5: problems 4,5; Homework 6: problems 2,3,4,5; Homework 7: all problems;
Homework 8: all problems.

Happy studying! I hope you all get As!