1. (a) \( \Delta x = 4/3 \), hence \( \text{MID}(3) = \left( \frac{1}{1+2/3} + \frac{1}{1+6/3} + \frac{1}{1+10/3} \right) (4/3) = 1.5512 \)
(b) \( \text{LEFT} = (-4 - 2.25 - 1 - 0.25)0.5 = -3.75 \) and \( \text{RIGHT} = (-2.25 - 1 - 0.25 - 0)0.5 = -1.75 \). Hence \( \text{TRAP} = -2.75 \).

2. (a) \( \text{MID} = (1.492 + 2.48 + 2.92 + 2.98)0.5 = 4.936 \).
(b) \( \text{TRAP} = (3.915 + 5.345)/2 = 4.63 \Rightarrow \text{SIMP} = (2 \cdot 4.936 + 4.63)/3 = 4.834 \).

3. (a) \( \text{RIGHT}, \text{MID}, \text{TRAP}, \text{LEFT} \). Reason: since the function is decreasing, we have \( \text{RIGHT} < \text{LEFT} \). Also, \( \text{TRAP} \) is the average of \( \text{RIGHT} \) and \( \text{LEFT} \). In addition, the function is concave up, which implies \( \text{MID} < \text{TRAP} \).
(b) Errors: exact - right(5) = .0032, exact - mid(5) = .0012, exact - trap(5) = -.0005, exact - left (5) = -.0043.
(c) \( \text{LEFT} \) and \( \text{RIGHT} \): the first 3 decimals will be correct since the error improves by 1 decimal place. \( \text{TRAP} \): the first 5 decimals will be correct since the error is improved by 2 decimal places.

4. (a) Improper. \( \int_{1}^{\infty} \frac{3}{\sqrt{2 + x}} \, dx = \lim_{b \to \infty} \int_{1}^{b} 3(2 + x)^{-1/2} \, dx = \lim_{b \to \infty} 6(2 + x)^{1/2} \bigg|_{1}^{b} = \lim_{b \to \infty} 6\sqrt{2 + b} - 6\sqrt{3} = \infty \). Hence the integral diverges.
(b) Improper. \( \int_{-\infty}^{5} \frac{2}{2x + 2} \, dx = \lim_{c \to -1+} \int_{c}^{5} \frac{1}{x + 1} \, dx = \lim_{c \to -1+} \ln |x + 1| \bigg|_{c}^{5} = \lim_{c \to -1+} \ln 6 - \ln |c + 1| = \infty \). Hence the integral diverges.
(c) Improper. \( \int_{0}^{5} \frac{2}{t^2 + 3t} \, dt = \lim_{c \to -0+} \int_{c}^{5} \frac{2}{t(t + 3)} \, dt = \lim_{c \to -0+} \int_{c}^{5} \frac{2/3}{t} - \frac{2/3}{t + 3} \, dt = \lim_{c \to -0+} \frac{2}{3} \ln |t| - \frac{2}{3} \ln |t + 3| \bigg|_{c}^{5} = \lim_{c \to -0+} \frac{2}{3} \ln \frac{5}{8} - \frac{2}{3} \ln \frac{c}{c + 3} = \infty \). Hence the integral diverges.
(d) Improper. \( \int_{-\infty}^{\infty} e^{3t} \, dt = \int_{-\infty}^{0} e^{3t} \, dt + \int_{0}^{\infty} e^{3t} \, dt = (I) + (II) \).
We now analyze \( (I) \) and \( (II) \) separately:

\( (I) = \lim_{c \to -\infty} \int_{c}^{0} e^{3t} \, dt = \lim_{c \to -\infty} \frac{1}{3} e^{3t} \bigg|_{c}^{0} = \lim_{c \to -\infty} \frac{1}{3} - \frac{1}{3} e^{c} = \frac{1}{3} - 0 \). Hence \( (I) \) converges.

\( (II) = \lim_{b \to -\infty} \int_{b}^{0} e^{3t} \, dt = \lim_{b \to -\infty} \frac{1}{3} e^{3t} \bigg|_{b}^{0} = \lim_{b \to -\infty} \frac{1}{3} e^{b} - \frac{1}{3} = \infty \). Hence \( (II) \) diverges.

We conclude that \( \int_{-\infty}^{\infty} e^{3t} \, dt \) also diverges.

5. (a) “Behaves-like” analysis: \( \frac{x}{\sqrt{1 + x^6}} \approx \frac{x}{\sqrt{x^6}} = \frac{1}{x^2} \) when \( x \) is large (\( p=2 \)). Hence we suspect convergence. We now compare the integrand with a larger function whose integral converges. We note that \( \sqrt{1 + x^6} \geq \sqrt{x^6} = x^3 \) for \( x \geq 1 \), which implies that the following inequality is valid: \( 0 \leq \frac{x}{\sqrt{1 + x^6}} \leq \frac{x}{\sqrt{x^6}} = \frac{1}{x^2} \), for \( 1 \leq x < \infty \). We conclude from the comparison test that \( \int_{1}^{\infty} \frac{x}{\sqrt{1 + x^6}} \, dx \) converges.
(b) "Behaves-like" analysis: \( \frac{t^2 + 1}{t^2 - 1} \approx \frac{t^2}{t^2} = 1 \) for large \( t \) (p=0). Hence we suspect divergence. We now compare the integrand with a smaller function whose integral diverges; For this we note that \( t^2 < t^2 + 1 \) and that \( t^2 > t^2 - 1 \), which imply that the following inequality is valid: \( 1 = \frac{t^2}{t^2} \leq \frac{t^2 + 1}{t^2 - 1} \) for \( 2 \leq t < \infty \). We conclude from the comparison test that \( \int_2^{\infty} \frac{t^2 + 1}{t^2 - 1} \, dt \) diverges.

6. By taking sections perpendicular to the axis of rotation, we get "washers". At the tickmark \( x_j \) the washer has inner radius \( r_j = 2x_j^2 \), outer radius \( R_j = 1 \), and thickness \( \Delta x \). The sum that approximates the volume is

\[
V \approx \sum_{j=0}^{n} (\pi R_j^2 - \pi r_j^2) \Delta x = \sum_{j=0}^{n} (\pi 1^2 - \pi (2x_j^2)^2) \Delta x
\]

The exact volume is obtained by taking limit as \( \Delta x \to 0 \). We have,

\[
Vol(S) = \int_{0}^{\sqrt{2}/2} (\pi - \pi 4x^4) \, dx = \frac{2\sqrt{2}\pi}{5} \approx 1.777153
\]

7. By taking sections perpendicular to the axis of rotation, we get "disks". At the tickmark \( y_j \) the radius is \( r_j = x_j = \sqrt{y_j/2} \) and the thickness is \( \Delta y \).

The sum that approximates the volume is

\[
\sum_{j=0}^{n} \pi r_j^2 \Delta y = \sum_{j=0}^{n} \pi (\sqrt{y_j/2})^2 \Delta y = \sum_{j=0}^{n} \pi y_j/2 \Delta y
\]

The volume is obtained by taking limit as \( \Delta y \to 0 \). We have,

\[
Vol(S) = \int_{0}^{1} \frac{\pi}{2} y \, dy = \frac{\pi}{4}
\]

8. By taking sections perpendicular to the axis of rotation, we get "washers". At the tickmark \( y_j \) the washer has inner radius \( r_j = 1 \), outer radius \( R_j = 1 + \sqrt{y_j/2} \), and thickness \( \Delta y \). The sum that approximates the volume is

\[
V \approx \sum_{j=0}^{n} (\pi R_j^2 - \pi r_j^2) \Delta y = \sum_{j=0}^{n} (\pi (1 + \sqrt{y_j/2})^2 - \pi 1^2) \Delta y
\]

The volume is obtained by taking limit as \( \Delta x \to 0 \). We have,

\[
Vol(S) = \int_{0}^{1} (\pi (1 + \sqrt{y/2})^2 - \pi) \, dy = \pi (\frac{1}{4} + \frac{2\sqrt{2}}{3}) \approx 3.74732
\]
9. a) 
\[
\sum_{j=0}^{n} \frac{0.004}{1 + r_j^2} 2\pi r \Delta r
\]
b) 
\[
\int_{0}^{7000} \frac{(0.004)2\pi r}{1 + r^2} \Delta r
\]
10. a) 
\[
\sum_{j=0}^{n} (2 + 0.015x)\Delta x
\]
b) 
\[
\int_{0}^{1} (2 + 0.015x)dx = 2.0075
\]
c) 
\[
\bar{x} = \frac{\int_{0}^{1} x (2 + 0.015x)dx}{\int_{0}^{1} (2 + 0.015x)dx} = \frac{1.0050}{2.0075} = 0.50062266
\]
11. A plot of \( y = \frac{3 - x}{1 + x} \) produced with a graphing calculator shows that the plate has the shape shown in the figure. It is clear that the curve meets the X and Y axes at \( x = 3 \) and \( y = 3 \) respectively. Since the plate has constant density, the formulas in page 365 of the text apply.

The total mass of the plate is 
\[
Mass = \int_{0}^{3} 0.15 \frac{3-x}{1+x} dx \approx 0.381777
\]
The center of mass \((\bar{x}, \bar{y})\) is given by
\[
\bar{x} = \frac{\int_{0}^{3} x 0.15 \frac{3-x}{1+x} dx}{Mass} = \frac{0.293223}{0.381777} \approx 0.768062
\]
By symmetry we have that \( \bar{y} = \bar{x} = 0.768062 \). Hence \((\bar{x}, \bar{y}) = (0.768062, 0.768062)\).

Note: to obtain \( \bar{y} \) with a calculation, it may be done as follows:
Solve for \( x \) in \( y = \frac{3 - x}{1 + x} \) to obtain \( x = \frac{3 - y}{1 + y} \). Then,
\[
\bar{y} = \frac{\int_{0}^{3} y 0.15 \frac{3-y}{1+y} dy}{Mass} = \frac{0.293223}{0.381777} \approx 0.768062
\]
12. Slice the drum with horizontal, circular sections. Each section corresponds to a tickmark \( y_j \) on the vertical axis. The number of bacteria in a section at tickmark \( y_j \) is
\[
Number(Section_j) \approx \text{density} \cdot \text{volume} = (3.5 - 0.05y_j) \pi (24)^2 \Delta y
\]
The total number of bacteria is approximated by the Riemann Sum
\[
Number(Container) = \sum_{j=1}^{n} (3.5 - 0.05y_j) \pi (24)^2 \Delta y
\]
The exact number is obtained by passing to the limit as $\Delta y \to 0$. It is,

$$\int_0^{36} (3.5 - 0.05y) \pi (24)^2 dy = 169375 \text{ million bacteria}$$

13.

A cross-section of the cone (shown in the figure) is bounded by the lines $y = \pm 2x$ and $y = 20$. Introduce tick marks in the $y$-axis.

The slab $S_j$ at height $y_j$ is a disk with radius $R_j = x_j = y_j/2$ and thickness $\Delta y$, so its volume is $\pi (y_j/2)^2 \Delta y$, and its weight is $62.4 \pi (y_j/2)^2 \Delta y$. The work involved in raising the slab a distance of $(20 - y_j)$ to the top of the cone is

$$w_j = (20 - y_j) 62.4 \pi (y_j/2)^2 \Delta y$$

The total work is approximated by

$$W \approx \sum_{j=0}^{n} (20 - y_j) 62.4 \pi (y_j/2)^2 \Delta y$$

The exact work is given by

$$\int_0^{20} (20 - y) 62.4 \pi (y/2)^2 dy = 653451.2720$$

14. A sketch of the dam is shown in the figure below.

Note that the equation of the right hand, non-horizontal side is $y = x - 30$. Introduce tick marks $y_0, y_1, \ldots, y_n$, in the $y$ axis. At height $y_j$, the slab has area $\approx (y_j + 30) \Delta y$, and the pressure at this height is $62.4(30 - y_j)$. Therefore the force on the slab is

$$F_j = 62.4(30 - y_j)(y_j + 30) \Delta y$$

The total force is approximated by

$$F \approx \sum_{j=0}^{n} 62.4(y_j + 30)(30 - y_j) \Delta y$$
The exact value of the total force is obtained by taking the limit as $\Delta y \to 0$:

$$F = \int_0^{30} 62.4(y_j + 30)(30 - y_j)dy = 1, 123, 200$$

15. a) $\int_1^7 \frac{2}{x^2} dx = 0.1569$
   b) $\int_{1.5}^2 \frac{2}{x^2} dx = \frac{1}{3}$

16. a) $\int_1^T \frac{2}{x^2} dx = 0.5 \Rightarrow \left. -2x \right|_1^T = 0.5 \Rightarrow \frac{-2}{T} + 2 = 0.5 \Rightarrow T = \frac{4}{3}$
   
   \[ \bar{x} = \int_1^T x \frac{2}{x^2} dx = \int_1^T \frac{2}{x} dx = 2 \ln(2) \]
   
   b) $\int_0^T 2e^{-2x} dx = 0.5 \Rightarrow \left. -e^{-2x} \right|_0^T = 0.5 \Rightarrow -e^{-2T} + 1 = 0.5 \Rightarrow T = \frac{\ln(0.5)}{-2} \approx 0.346574$
   
   \[ \bar{x} = \int_0^\infty x \cdot 2e^{-2x} dx = \lim_{b \to \infty} \int_0^b 2x e^{-2x} dx = \lim_{b \to \infty} \left( -\frac{x}{e^{2x}} + \frac{1}{2e^{2b}} \right) \bigg|_0^b = \lim_{b \to \infty} \left( -\frac{b}{e^{2b}} + \frac{1}{2e^{2b}} \right) - \left( 0 - \frac{1}{2} \right) = \frac{1}{2} \]

17. a) The cumulative distribution function is an antiderivative of the density, so $P = \int \frac{2}{x^2} dx = -\frac{2}{x} + c$. We now find $c$. We also know that $P = 0$ when $x = 1$ (first value of $x$). Substituting we find $c = 2$, so $P(x) = -\frac{2}{x} + 2$.

Another way to solve it: $P(x) = \int_1^x \frac{2}{t^2} dt = -\frac{2}{t} \bigg|_1^x = -\frac{2}{x} + 2$.

b) $P = \int 2e^{-2x} dx = -e^{-2x} + C$. We also know that $P = 0$ when $x = 0$. Substituting into $P$ we have $0 = -1 + C$, that is, $C = 1$. We conclude that $P = -e^{-2x} + 1$.

Another way to solve it: $P(x) = \int_0^x 2e^{-2t} dt = -e^{-2t} \bigg|_0^x = -e^{-2x} + 1$.

18. The increasing function is the Cumulative Distribution Function, which we know has range from 0 to 1. This gives the vertical range, so the tick marks on the $y$ axis are at \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}. Also, we know that the region under the density function (approx. 2.5 rectangles) has area 1. Then each rectangle has area $1/2.5 = 0.4$. But we also know the height of the rectangle is 0.2. Then the base of the rectangle is approximately 0.4/0.2 = 2. Therefore the tick marks on the x-axis are at \{0, 2, 4, 6, 8, 10\}.