MTH 142 Practice Problems for Exam 1  Last updated 2/6/2001

This is a selection of sample problems. The actual test will have fewer problems.
Sections 7.1 – 7.8

Part I. No Calculators

You may not have a calculator while this part of the test is in your posession. When you are through with this part, hand it in. Then you may work on the rest of the test using a calculator.

(1.) \( \int \frac{6t + 1}{t^3 + 2t^2 + t} \) dx  
(2.) \( \int \frac{\ln(x^3)}{x} \) dx  
(3.) \( \int_0^1 (-3x + 2)e^{2x} \) dx

(4.) \( \int e^{\sin^2 x} \sin x \cos x \) dx  
(5.) \( \int \frac{x^2}{2x - 3} \) dx  
(6.) \( \int \frac{6x - 4}{x^2 + 8x - 9} \) dx

(7.) \( \int \frac{-2}{x^2 + x + \frac{26}{4}} \) dx  
(8.) \( \int \sin(e^x)e^{2x} \) dx  
(9.) \( \int_1^3 \frac{t^2}{t^2 + 9} \) dt

(10.) \( \int \frac{1}{1 + \sqrt[3]{x}} \) dx  
(11.) \( \int \ln(2t) \) dt  
(12.) \( \int x \sin(wt) \) dt
Part II. Calculators are allowed

13. Use the midpoint rule with \( n = 3 \) to approximate \( \int_0^1 \frac{1}{1+x} \, dx \). Do the calculations by hand, write down details.

14. The following table gives values of a function \( f \), whose concavity does not change in the interval \([0,2]\):

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>1.75</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0.0</td>
<td>1.492</td>
<td>2.08</td>
<td>2.48</td>
<td>2.75</td>
<td>2.92</td>
<td>3.00</td>
<td>2.98</td>
<td>2.86</td>
</tr>
</tbody>
</table>

We want to estimate \( \int_0^2 f(x) \, dx \).

a) Find the midpoint estimate with 4 subdivisions, MID(4).

b) You can easily calculate that LEFT(4) = 3.915 and RIGHT(4) = 5.345. Find the trapezoid and Simpson’s estimates TRAP(4) and SIMP(4).

c) Determine the concavity of the function on the interval \([0,2]\) by plotting points or by using numerical calculations.

d) Explain why your value of SIMP(4) must be approximately within 0.33 of the exact value of the integral. (Hint: How far apart are your values for TRAP(4) and MID(4)?)

15. The numerical approximation of an integral \( \int_a^b f(x) \, dx \) with LEFT(n), RIGHT(n), TRAP(n), and MID(n), produced the numbers shown below (not necessarily in the same order):

6.725, 6.745, 6.762, 6.800

It is known that the function \( f(x) \) is decreasing and concave up. Choose a suitable method for each number. Give a reason for your choice.

16. The exact value of an integral \( \int_a^b f(x) \, dx \) is 2.50. It is also known that LEFT(5) = 2.532431 and MID(5) = 2.502215.

a) What is the error in each case.

b) If only one decimal is correct in both figures, how many decimals you predict will be correct when using MID(50) to calculate the integral? Explain.

Say which of the following integrals are improper. For those that are improper, determine if they are convergent or divergent. Use the comparison test whenever appropriate.

\[
\begin{align*}
(18.) \ & \int_1^\infty \frac{x}{\sqrt{1+x^5}} \, dx \\
(19.) \ & \int_{-2}^5 \frac{x^2}{x+1} \, dx \\
(20.) \ & \int_0^5 \frac{2}{t^2 + 3t} \, dt \\
(21.) \ & \int_0^\infty te^{-t} \, dt \\
(22.) \ & \int_1^\infty \frac{t}{t^2 + 1} \, dt \\
(23.) \ & \int_0^1 t^2 \sin t \, dt
\end{align*}
\]