MTH307 - HOMEWORK 5

Solutions to the questions in Section B should be submitted by the start of class on 10/25/18.

A. Warm-up Questions

Question A.1. Prove the following.

(i) $\{n \in \mathbb{Z} \mid 9 \mid n\} \subseteq \{x \in \mathbb{Z} \mid 3 \mid x\}$.
(ii) $\{6n \mid n \in \mathbb{Z}\} = \{2n \mid n \in \mathbb{Z}\} \cap \{3y \mid y \in \mathbb{Z}\}$.
(iii) If $p, q \in \mathbb{N}$ then $\{pn \mid n \in \mathbb{N}\} \cap \{qn \mid n \in \mathbb{N}\} \neq \varnothing$.
(iv) $\{4^q \mid q \in \mathbb{Q}\} = \{2^q \mid q \in \mathbb{Q}\}$.
(v) $\{4^n \mid n \in \mathbb{Z}\} \subseteq \{2^n \mid n \in \mathbb{Z}\}$.

Question A.2. Let $\mathcal{U} = \{n \in \mathbb{Z} \mid 1 \leq n \leq 9\}$ be the universal set and $A = \{1, 5, 9\}$, $B = \{2, 3, 5, 7\}$, $C = \{1, 2, 3, 5, 8, 9\}$ and $D = \{2, 4, 6, 8\}$. Compute the following.

(i) $A \cup B$.
(ii) $A \cup (B \cap C)$.
(iii) $(A \cup B) \cap C$.
(iv) $(A \times B) - (C \times D)$.
(v) $\overline{C \cup D}$.
(vi) $\overline{C} \cup \overline{D}$.
(vii) $(C - A) \cap D$.
(viii) $(D \cup A) - (C \cap B)$.

Question A.3. Let A, B, C and D be subsets of the universal set $\mathcal{U}$. Prove the following.

(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
(iii) $A \cup (B \cup C) = (A \cup B) \cup C$.
(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
(v) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
(vi) $A \cup \overline{A} = \mathcal{U}$.
(vii) $A - B = A \cap \overline{B}$.
(viii) $(A \cup B) - C \subseteq (A - C) \cup B$.

Question A.4. Let A, B, C and D be sets. Prove or disprove the following.

(i) If $x \notin B$ and $A \subseteq B$ then $x \notin A$.
(ii) If $A \subseteq B \cup C$ then $A \subseteq B$ or $A \subseteq C$.
(iii) $A \subseteq B$ if and only if $A \cap B = A$.
(iv) If $A = B - C$ then $B = A \cup C$.
(v) If $A - B = \varnothing$ then $B \neq \varnothing$.
(vi) $(A \cup B) - B \subseteq A$.
(vii) $(A \cup B) - B = A$.
(viii) $A - (B - C) = (A - B) - C$.
(ix) If $A \subseteq C$ and $B \subseteq D$ then $A \cup B \subseteq C \cup D$.
(x) If $A \subseteq C$ and $B \subseteq D$ then $A - D \subseteq C - B$.
(xi) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

B. Submitted Questions

Question B.1. Let A, B and C be sets.

(i) Prove or disprove that $A \subseteq B \cap C$ if and only if $A \subseteq B$ and $A \subseteq C$.
(ii) Prove that $(A \cup B) - C = (A - C) \cup B$ if and only if $B \cap C = \varnothing$.

C. Challenge Questions

Question C.1. *Symmetric difference.* Given two sets A and B, we define the symmetric difference of A and B to be $A \triangle B = (A - B) \cup (B - A)$. Prove the following.

(i) Prove that $A \triangle B = (A \cup B) - (A \cap B)$.
(ii) Prove that $A \triangle B = B \triangle A$.
(iii) Prove that $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

Question C.2. *Ordered Pairs.* Formally, we may define an ordered pair as a set by $(x, y) = \{\{x\}, \{x, y\}\}$. Prove that $(x, y) = (z, w)$ if and only if $x = z$ and $y = w$ (and so this definition is consistent with our understanding of equality of ordered pairs).