MTH307 - HOMEWORK 2

Solutions to the questions in Section B should be submitted by the start of class on 09/27/18. You may like to remind yourself on the properties of integers and real numbers found on pages 23 and 27 respectively of the textbook.

A. Warm-up Questions

Question A.1. Prove the following.

(i) If x is odd then $x^2 + 3x + 9$ is odd.
(ii) If $x \in \mathbb{R}$ then $x^2 \geq 0$ (Use cases).
(iii) If $x \in \mathbb{Z}$ then $5x^2 + 3x + 7$ is odd.
(iv) If $a \mid b$ and $c \mid d$ then $ac \mid bd$.
(v) If $x < 0$ then $x + \frac{1}{x} \leq -2$.
(vi) If $0 < a < b$ then $0 < a^2 < b^2$.
(vii) If x is odd then x^3 is odd.
(viii) If $x \in \mathbb{R}$ and $x \neq 0$ then there exists a unique $y \in \mathbb{R}$ such that $xy = 1$.
(ix) If m and n are even then $4 \mid mn$.
(x) If $x, y \in \mathbb{R}$ then $\sqrt{xy} \leq \frac{x+y}{2}$.
(xi) There exists a prime number p with $90 < p < 100$.

B. Submitted Questions

Question B.1. Prove the following.

(i) Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$ then $x = y$ or $x + y = 5$
(ii) If $x \in \mathbb{R}$ and $0 < x < 4$ then $\frac{4}{x(4-x)} \geq 1$.

C. Challenge Questions

Question C.1. Prove the following.

(i) If $n^2 \mid n$ then $n = 1$ or $n = 0$ or $n = -1$.
(ii) If $x, y \in \mathbb{R}$ and $xy = 0$ then $x = 0$ or $y = 0$.
(iii) If $a, b, c \in \mathbb{R}$ then $ax^2 + bx + c = 0$ has a (real) solution u if and only if $b^2 - 4ac \geq 0$ and $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
(iv) If m is an odd integer, then there exists $k \in \mathbb{Z}$ such that $m^2 = 8k + 1$.