MTH 142 – Practice Exam – Chapters 9-11

Calculus II With Analytic Geometry
Fall 2011 - University of Rhode Island

- This practice exam is intended to help you prepare for the final exam for MTH 142 – Calculus II.
- This practice exam is **NOT** intended to be your only resource in preparing for the exam. Do **NOT** use this in place of your course notes, previous exams, quizzes, homework, and online homework.
- Questions that do not appear on this practice exam may appear on the actual final exam, and just because a question appears on this practice exam does not guarantee a similar question will appear on the actual final exam.
- Also note that this practice exam is not intended to give you an idea of the length of the actual exam.

Good luck!

1. For the sequence $s_n = 1 - n^2$ (not a series, the sequence!), choose the option that describes its behavior as $n \to \infty$.

   (a) Converges to 0, and all s_n are positive values
   (b) Diverges to $-\infty$
   (c) Converges to 0, and s_n takes on both positive and negative values
   (d) Diverges to $+\infty$
   (e) Converges to 1

---

2. For the sequence $s_n = \cos(1/n)$ (not a series, the sequence!), choose the option that describes its behavior as $n \to \infty$.

   (a) Converges to 0, and all s_n are positive values
   (b) Converges to 0, and s_n takes on both positive and negative values
   (c) Converges to 1
   (d) Diverges to $-\infty$
   (e) Diverges to $+\infty$

---

3. Find the sum of the series $\sum_{n=0}^{\infty}(3/2^n)$ if it converges.

   (a) 3
   (b) 6
   (c) Diverges
   (d) 9
   (e) 2

---

4. Find c such that $\sum_{n=0}^{\infty} c^n$ converges to 10.

   (a) 11/10
   (b) 9/10
   (c) No such c exists
   (d) 10/9
   (e) 10/11

---

5. Classify $\sum_{n=2}^{\infty} \frac{n^{3/2}}{n^2 - 1}$ as convergent or divergent, and give a correct reason for your answer.

   (a) Divergent: $\lim_{n\to\infty} a_n \neq 0$.
   (b) Divergent: Comparison test with the harmonic series.
   (c) Convergent: $\lim_{n\to\infty} a_n = 0$.
   (d) Convergent: Geometric series with ratio $1/2 < 1$.
   (e) Divergent: Comparison test with a geometric series.

---

6. Classify $\sum_{n=1}^{\infty} \frac{1}{n \cdot 5^n}$ as convergent or divergent, and give a correct reason for your answer.

   (a) Divergent: Comparison test with the harmonic series.
   (b) Convergent: Comparison test $\sum_{n=1}^{\infty} \frac{1}{n^5}$.
   (c) Convergent: Comparison test with $\sum_{n=1}^{\infty} (1/5)^n$.
   (d) Convergent: $\lim_{n\to\infty} a_n = 0$.
   (e) Divergent: $\lim_{n\to\infty} a_n \neq 0$.

---

7. Classify $\sum_{n=1}^{\infty} \frac{3^n + 4^n}{e^n}$ as convergent or divergent and give a correct reason for your answer.

   (a) Divergent: $\lim_{n\to\infty} a_n \neq 0$.
   (b) Divergent: constant multiple of the harmonic series.
   (c) Convergent: sum of two convergent geometric series.
   (d) Convergent: Comparison test with geometric series with ratio $e/4$.
   (e) Convergent: Comparison test with geometric series with ratio $3/e$.

---

8. Classify $\sum_{n=1}^{\infty} \frac{3^n}{(n+1)!}$ as convergent or divergent, and give a correct reason for your answer.

   (a) Convergent: Ratio test with limiting ratio 0.
   (b) Convergent: $\lim_{n\to\infty} a_n = 0$.
   (c) Divergent: Comparison test with the geometric series $\sum_{n=1}^{\infty} 3^n$.
   (d) Divergent: Ratio test with limiting ratio 3.
   (e) Divergent: $\lim_{n\to\infty} a_n \neq 0$.

---

9. Classify $\sum_{n=1}^{\infty} \frac{(n+3)!}{n^2 \cdot 3^n}$ as convergent or divergent, and give a correct reason for your answer.

   (a) Convergent: $\lim_{n\to\infty} a_n = 0$.
   (b) Divergent: Ratio test with limiting ratio ∞.
   (c) Convergent: Ratio test with limiting ratio 1/3.
   (d) Convergent: Comparison test with the geometric series $\sum_{n=1}^{\infty} (1/3)^n$.
   (e) Divergent: Integral test.

---

10. Find the radius of convergence of $\sum_{n=1}^{\infty} x^{n+1} 2(n+1)$.

   (a) 1
   (b) $1/\sqrt{2}$
   (c) $\sqrt{2}$
   (d) 2
   (e) 1/2

---

11. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n}{3^n}(x-1)^n$.

   (a) ∞
   (b) $\frac{3n}{n+1}$
   (c) $\frac{n+1}{3n}$
   (d) 3
   (e) 1

---

12. Find the interval of convergence, including any endpoints, of the series $\sum_{n=0}^{\infty} x^{2n}$.

(a) $-2 \le x \le 2$
(b) $-1 \le x \le 1$
(c) $-1 < x \le 1$
(d) $-1 < x < 1$
(e) $-1 \le x < 1$

---

13. Find the interval of convergence, including any endpoints, of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2}$.

(a) $-\infty < x < \infty$
(b) $1 < x < 3$
(c) $-1 \le x \le 1$
(d) $-1 < x < 1$
(e) $1 \le x \le 3$

---

14. Find the interval of convergence, including any endpoints, of the series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n}$.

(a) $-4 < x \le -2$
(b) $-4 \le x < -2$
(c) $2 \le x < 4$
(d) $-4 \le x \le -2$
(e) $2 < x \le 4$

---

15. Find the interval of convergence, including any endpoints, of the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{n}$.

(a) $4 < x \le 6$
(b) $-\infty < x < \infty$
(c) $4 < x < 6$
(d) $4 \le x \le 6$
(e) $4 \le x < 6$

---

16. Find the Taylor polynomial of degree 3 for xe^{-x} about $x = 0$.

(a) $x - x^2 + x^3/2$
(b) $x + x^2/2 + x^3/6$
(c) $1 - x + x^2/2 - x^3/6$
(d) $x + x^2 + x^3/2$
(e) $x - x^2/2 + x^3/3$

---

17. Find the Taylor polynomial $P_3(x)$ for $f(x) = e^{(x^2)}$ about $x = 0$.

(a) $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6}$
(b) $1 + x + 2x^2 + 3x^3$
(c) $1 + x + x^2$
(d) $1 + x^2$
(e) $1 + x^2 + x^3$

---

18. Find the binomial series for $f(x) = (1 + x)^{1/2}$ about $x = 0$.

(a) $\dfrac{x}{2} - \dfrac{x^2}{4} + \dfrac{3x^3}{8} - \dfrac{5x^4}{16} + \cdots$
(b) $1 - \dfrac{x}{2} + \dfrac{x^2}{8} - \dfrac{x^3}{16} + \cdots$
(c) $1 - \dfrac{x}{2} + \dfrac{x^2}{4} - \dfrac{3x^3}{8} + \cdots$
(d) $1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \cdots$
(e) $1 + \dfrac{x}{2} - \dfrac{x^2}{4} + \dfrac{3x^3}{8} - \cdots$

---

19. Find the binomial series representing $\left(1 + \dfrac{x}{2}\right)^{5/2}$ near $x = 0$.

(a) $1 + \dfrac{5x}{4} + \dfrac{15x^2}{32} + \dfrac{5x^3}{128} + \cdots$
(b) $1 + \dfrac{5x}{2} + \dfrac{15x^2}{16} + \dfrac{3x^3}{16} + \cdots$
(c) $1 + \dfrac{5x}{2} + \dfrac{15x^2}{4} + \dfrac{45x^3}{4} + \cdots$
(d) $1 + \dfrac{5x}{4} + \dfrac{15x^2}{8} + \dfrac{45x^3}{32} + \cdots$
(e) $1 + \dfrac{5x}{2} + \dfrac{15x^2}{4} + \dfrac{15x^3}{8} + \cdots$

---

20. Give the formula for the elementary function represented by the series $1 + x^2 + x^4 + x^6 + x^8 + \cdots$ within its interval of convergence.

(a) $1/(1 - x^2)$
(b) $1/(1 + x^2)$
(c) $[1/(1 - x)]^2$
(d) $x^2(1 - x)$
(e) $[1/(1 + x)]^2$

---

21. Find the sum of the series $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.

(a) ∞
(b) $\sin(3)$
(c) $\cos(3)$
(d) e^3
(e) $3e$

---

22. Give the formula for the elementary function represented by the series $x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots$ within its interval of convergence.

(a) $e^{(x^2)}$
(b) $x^2/(1+x)$
(c) xe^x
(d) x^2e^x
(e) $x^2/(1-x)$

---

23. Give the formula for the elementary function represented by the series $-x^2 + \frac{x^4}{3!} - \frac{x^6}{5!} + \frac{x^8}{7!} - \cdots$ within its interval of convergence.

(a) $-\cos(x^2)$
(b) $-x(\sin x)$
(c) $x^2\cos(x)$
(d) $e^{(-x^2)}$
(e) $-\sin(x^2)$

---

24. Find the Taylor series for e^{2x} about $x = 0$.

(a) $1 + 2x + x^2 + (1/3)x^3 + \cdots$
(b) $1 + 4x^2 + 16x^4 + 64x^6 + \cdots$
(c) $1 + 2x + 4x^2 + 8x^3 + \cdots$
(d) $1 + 2x + 2x^2 + (4/3)x^3 + \cdots$
(e) $1 + 2x^2 + (2/3)x^4 + (4/45)x^6 + \cdots$

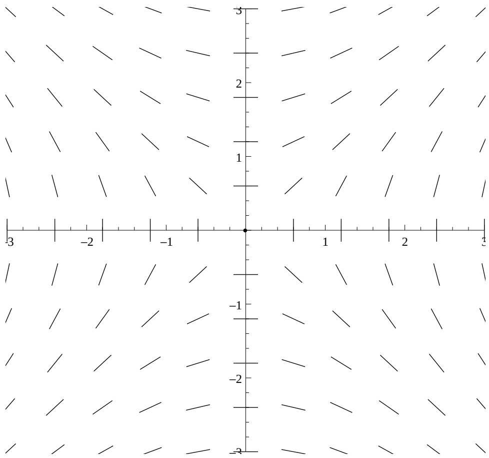
---

25. Find the Taylor series for $x^3\cos x$ about $x = 0$.

(a) $x^3 + x^4 + x^6/2 + x^7/6 + \cdots$
(b) $x^3 - x^5/2! + x^7/4! - x^9/6! + \cdots$
(c) $x^3/3! + x^4/4! + x^6/6! + x^7/7! + \cdots$
(d) $x^3/3! - x^5/5! + x^7/7! - x^9/9! + \cdots$
(e) $1 - x^6/2! + x12/4! - x^{18}/6! + \cdots$

---

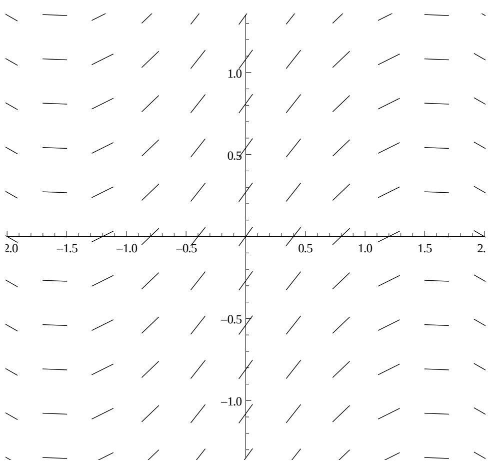
26. Select the differential equation below corresponding to the slope field pictured.



(a) $\frac{dy}{dx} = x^2 - 1$
(b) $\frac{dy}{dx} = y^2 - x^2$
(c) $\frac{dy}{dx} = y + 1$
(d) $\frac{dy}{dx} = x/y$
(e) $\frac{dy}{dx} = xy$

---

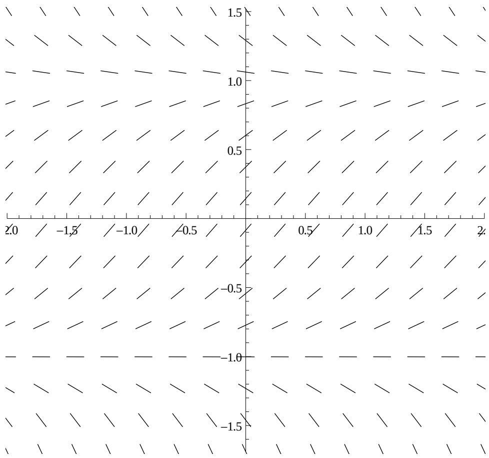
27. Select the differential equation below corresponding to the slope field pictured.



(a) $\frac{dy}{dx} = \cos(x)$
(b) $\frac{dy}{dx} = xy$
(c) $\frac{dy}{dx} = y^2 - 1$
(d) $\frac{dy}{dx} = x - y$
(e) $\frac{dy}{dx} = -x^2$

---

28. Select the differential equation below corresponding to the slope field pictured.



(a) $\frac{dy}{dx} = y(1-y)$

(b) $\frac{dy}{dx} = (1-x)(1+y)$

(c) $\frac{dy}{dx} = (1-y)(1+y)$

(d) $\frac{dy}{dx} = -x/y$

(e) $\frac{dy}{dx} = y^2 - 1$

---

29. Consider the differential equation $dy/dx = x - y$. Let $y = f(x)$ be the solution of this equation containing the point $(0, 2)$. Estimate $f(1)$ using Euler's method with $\Delta x = 1/2$.

(a) 19/4

(b) 15/4

(c) 1/2

(d) 1

(e) 3/4

---

30. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x+y}{y}$. Let $y = f(x)$ be the solution of this equation containing the point $(0, 1)$. Estimate $f(1/2)$ using Euler's method with $\Delta x = 1/4$.

(a) 17/10

(b) 27/20

(c) 13/10

(d) 49/20

(e) 31/20

---

31. Solve the differential equation $dy/dx = y^2 \sin 2x$.

(a) $-2/y = (\cos 2x) + C$
(b) $1/y = (\cos 2x) + C$
(c) $2/y = (\cos 2x) + C$
(d) $y^3/3 = -(\cos 2x)/2 + C$
(e) $-1/y = (\cos 2x) + C$

---

32. Solve the differential equation $dy/dx = \sqrt{x}\sqrt{y}$.

(a) $-\sqrt{y} = (1/3)x^{3/2} + C$
(b) $-1/\sqrt{y} = (2/3)x^{3/2} + C$
(c) $1/\sqrt{y} = (2/3)x^{3/2} + C$
(d) $\sqrt{y} = (1/3)x^{3/2} + C$
(e) $-\sqrt{y} = (2/3)x^{3/2} + C$

---

33. Find the solution of the differential equation $dy/dx = \sqrt{x+1}\sqrt{y+5}$ such that $y = 4$ when $x = 3$.

(a) $y = \sqrt{x+1}\sqrt{y+5} - 2$
(b) $2/\sqrt{y+5} = (1/3)(x+1)^{3/2} - 2$
(c) $1/\sqrt{y+5} = (1/3)(x+1)^{3/2} - 7/3$
(d) $\sqrt{y+5} = (1/3)(x+1)^{3/2} + 1/3$
(e) $\sqrt{y+5} = (2/3)(x+1)^{3/2} - 7/3$

---

34. Solve the initial value problem $dy/dx = y/x$, $y = 2$ when $x = 1$. Express your answer in the form $y = f(x)$.

(a) $y = e\sqrt{x}$
(b) $y = 2\sqrt{x}$
(c) $y = e^4x^2$
(d) $y = 2x$
(e) $y = e^2x$

---

35. Solve the initial value problem $dy/dx = y/x$, $y = 2$ when $x = 1$. Express your answer in the form $y = f(x)$.

(a) $y = 2x$
(b) $y = 2\sqrt{x}$
(c) $y = e^4x^2$
(d) $y = e^2x$
(e) $y = e\sqrt{x}$

Practice Exam Solutions

1.	B
2.	C
3.	B
4.	B
5.	B
6.	C
7.	A
8.	A
9.	B
10.	A
11.	D
12.	D
13.	E
14.	B
15.	A
16.	A
17.	D
18.	D
19.	A
20.	A
21.	D
22.	D
23.	B
24.	D
25.	B
26.	D
27.	A
28.	C
29.	E
30.	E
31.	C
32.	D
33.	D
34.	D
35.	A