

## mth 692 Probability on Discrete Structures: Spring 2009

### Program

1. The basic method: Ramsey numbers, the random graph,  $S_k$  property of tournaments, dominating sets in graphs, hypergraph 2-coloring
2. Linearity of expectation: sum-free sets, fixed points of random permutations, lower and upper bounds on the maximum number of hamiltonian paths in tournaments, balancing vectors. Existence of graphs with large girth and large chromatic number.
3. The second moment method: Chebyshev's inequality, Turan's proof of the Hardy Ramanujan theorem, random graphs and threshold functions, the threshold for a triangle  $K_3$ , thresholds of subgraphs. Monotone properties of graphs and 0-1 laws for the first order properties of graphs. Random graph evolution. Cliques in random graphs.
4. Bounding large deviations (Chernoff bounds), large deviation bounds on degrees in  $G(n, 1/2)$ . A lower bound on  $r(3, t) > c(t/\ln t)^2$  by Erdos, Erdos-Tetali disjointness lemma.
5. Lovasz Local Lemma (both the symmetric and general forms). Applications of Lovasz Local Lemma to: hypergraph 2-coloring, coloring of integers which makes all shifts of given set multicolored.
6. Tight concentration on product spaces. The size of the image of a random function. Concentration of the chromatic number of the random graph: theorem by Shamir and Spencer, the four point concentration of the chromatic number of sparse random graphs by Bollobás.  
Concentration of Lipschitz functions.
7. Martingales. The edge exposure martingale. Large deviation inequalities for martingales. Azuma's inequality. Concentration of 1-Lipschitz functions on  $S_n$ . Talagrand's inequality. Concentration of the largest eigenvalue of random matrices.
8. Additional topics: random regular graphs, random k-SAT, entropy, extremal and ramsey properties of random graphs, broadcasting on random graphs. Bipartite Ramsey numbers. Expanders and Ramanujan graphs. Random clique complexes. Percolation.