mth 548 / csc 548 Topics in Combinatorics – Graph Theory: Spring 2009

Program

[a] Fundamentals: basic definitions, isomorphism, subgraphs, vertex degrees, k-regular graphs, edge density. Walk, path, cycle, girth, diameter, central vertices. Vertex and edge connectivity.

Trees, spanning trees, Cayley's theorem.

Bipartite graphs, characterization of bipartite graphs vis odd cycles. Eulerian graphs.

Linear algebra tools, adjacency and incidence matrices, eigenvalues and basic bounds for them.

Other notions of graphs.

[b] Matchings: augmenting paths and theorem of Berge, Hall's theorem, vertex-covers, Konig-Egervary theorem for bipartite graphs, linear programming formulation of a maximal matching and a minimal vertex-cover. The relations between max size of an independent set, min size of a vertex cover, max size of a matching, and min size of an edge-cover. Gallai's theorem: $\alpha'(G) + \beta'(G) = |V(G)|$. Konig's theorem for bipartite graphs.

Tutte's 1-factor theorem. Every 3-regular graph without a cut-edge has a 1-factor.

[c] Connectivity: the structure of 2-connected graphs, the block graph, local and global versions of theorems by Menger and Fulkerson-Ford. Whitney's and Menger's theorems.

[d] Planar graphs: $K_5, K_{3,3}$ are not planar, Euler's formula, the number of edges in a connected planar graph, every connected planar graph contains a vertex of degree at most 5.

Subdivision of edges, topological minors, Kuratowski's theorem.

[e] Colorings: Vertex and edge coloring, chromatic and edge-chromatic numbers.

Coloring planar graphs, every planar graph is 5-colorable, 4CT,

Bounds on $\chi(G)$, greedy colorings, bounds on $\chi(G), \chi(\overline{G})$. Let G be a graph with a vertex partition $V_1 \cup \ldots \cup V_k$ with a missing edge between any pair of sets in the partition, then $\chi(G) \leq |V(G)| - k + 1$. Highly chromatic graphs of large girth.

k-critical graphs, the minimal degree of k-critical graphs, vertex cuts of k-critical graphs, theorem on k-critical graphs with a vertex cut of size two. Brooks' theorem.

Edge coloring, edge-chromatic number. For bipartite graphs $\chi'(G) = \Delta(G)$. Vizing's theorem. Graphs of class 1 and 2. If $e(G) > \Delta(G) \cdot \beta'(G)$, G is of class 2. Every overfull graph is of class 2. The edge chromatic number of complete graphs K_n.

[f] Extremal graph theory: Turan numbers $ex(n, H)$. Turan's number for paths $ex(n, P_k)$. Turan's graphs $T^r(n)$ and Turan's theorem for K^r ($T^r(n)$ is extremal for K^{r+1} and $ex(n, K^{r+1}) = |E(T^r(n))|$). A geometric application to Euclidean normed spaces. The Erdős-Stone theorem. $ex(n, H) = (1 - 1/(\chi(H) - 1))\binom{n}{2} + O(n^2)$.

Bipartite Turan numbers: upper bounds on $ex(n, K_{r,s})$, $ex(n, T)$ (T a tree), lower bounds on $ex(n, K_{2,2})$. Determining $ex(n, P_k)$, P_k a path of length k.

Szemerédi's regularity lemma. ϵ-regular pairs. A proof of the Erdos-Stone theorem.

[g] Introduction to Ramsey theory: Ramsey's theorem, lower and upper bounds for Ramsey numbers and their asymptotics. Multicolor ramsey numbers. Graph ramsey theory, $r(T_m, K_n) = (m-1)(n-1)+1$. Schur's theorem, applications.

[h] Hamiltonian graphs: Theorems of Dirac, Chvatal, and Ore. The closure of a graph.