An Atlas of Small Groups

Sara Day Eli Glatt Ben Salisbury

May 4, 2005

Abstract

This atlas contains information about ten groups of order n where $4 \leq n \leq 20$. We will include Cayley tables, basic properties, subgroup diagrams, and where each group naturally arises. We have chosen the following set of n: $\{4, 5, 6, 7, 9, 11, 13, 15, 17, 19\}$.

Introduction

Definition 1 *A* group *is a set together with an operation such that associativity, existence of an identity element, and existence of inverse elements are all satisfied.*

Definition 2 *H is a* subgroup *of G iff H is nonempty, $ab \in H$ for all $a, b \in H$, and $a^{-1} \in H$ for all $a \in H$.*

Definition 3 *S_n is the set of all permutations on the let $\{1, 2, \ldots, n\}$. S_n has order $n!$.*

Definition 4 *D_n is set of all isometries of a regular n-gon. $|D_n| = 2n$.*

Definition 5 *$\mathbb{Z}_n$ is complete set of congruence classes modulo n.*

Definition 6 *For the $\mathbb{Z}_n$ groups, the operation $\oplus$ is defined as follows:*

$$[a] \oplus [b] = [a + b] \text{ for all } [a], [b] \in \mathbb{Z}_n$$

Definition 7 *For the $\mathbb{Z}_n \times \mathbb{Z}_m$ groups, the operation $\oplus$ is defined as follows:*

$$\big([a_1], [b_1]\big) \oplus \big([a_2], [b_2]\big) = \big([a_1 + a_2], [b_1 + b_2]\big) \text{ for all } [a_1], [a_2] \in \mathbb{Z}_n \text{ and } [b_1], [b_2] \in \mathbb{Z}_m$$

Definition 8 *The operation $\circ$ is defined on S_n by*

$$\tau \circ \sigma = \tau(\sigma) \text{ for all } \tau, \sigma \in S_n$$

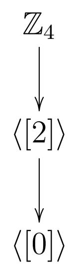
Definition 9 *The set of all integral powers of a, $\langle a \rangle = \{a^n : n \in \mathbb{Z}\}$.*

Definition 10 *An isomorphism is a map between two groups, $\varphi : G \to H$, such that φ is one-to-one and onto, and*

$$\varphi(xy) = \varphi(x)\varphi(y) \text{ for all } x, y \in G.$$

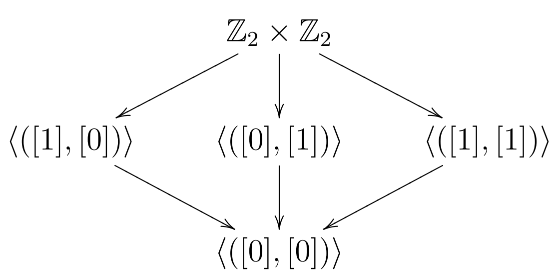
When $n = 4$, we get two nonisomorphic groups: $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$.

$\langle \mathbb{Z}_4, \oplus \rangle$	$[0]$	$[1]$	$[2]$	$[3]$
$[0]$	$[0]$	$[1]$	$[2]$	$[3]$
$[1]$	$[1]$	$[2]$	$[3]$	$[0]$
$[2]$	$[2]$	$[3]$	$[0]$	$[1]$
$[3]$	$[3]$	$[0]$	$[1]$	$[2]$



We can see that $\mathbb{Z}_4$ is cyclic with generators $[1]$ and $[3]$, so it is also abelian. This group is a permutation group.

$\langle \mathbb{Z}_2 \times \mathbb{Z}_2, \oplus \rangle$	$([0],[0])$	$([1],[0])$	$([0],[1])$	$([1],[1])$
$([0],[0])$	$([0],[0])$	$([1],[0])$	$([0],[1])$	$([1],[1])$
$([1],[0])$	$([1],[0])$	$([0],[0])$	$([1],[1])$	$([0],[1])$
$([0],[1])$	$([0],[1])$	$([1],[1])$	$([0],[0])$	$([1],[0])$
$([1],[1])$	$([1],[1])$	$([0],[1])$	$([1],[0])$	$([0],[0])$



$\mathbb{Z}_2 \times \mathbb{Z}_2$ is a cyclic group that is used in number theory.

There is just one group up to isomorphism of order 5.

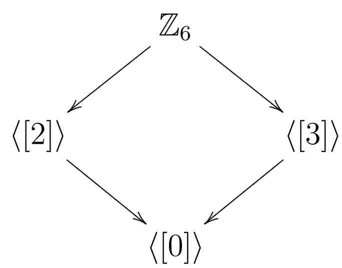
$\langle \mathbb{Z}_5, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

$$\begin{array}{c} \mathbb{Z}_5 \\ \downarrow \\ \langle [0] \rangle \end{array}$$

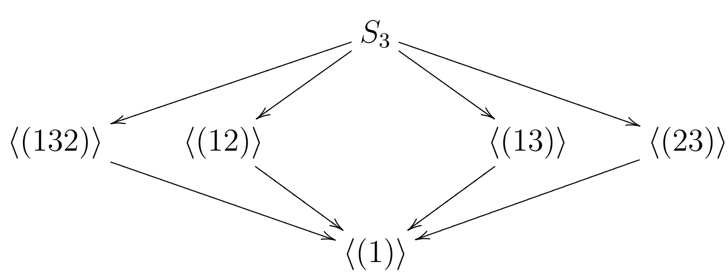
$\mathbb{Z}_5$ is cyclic and it is a permutation group. Every nonidentity of $\mathbb{Z}_5$ are generators of $\mathbb{Z}_5$. This is the only group of order 5 up to isomorphism.

There are two nonisomorphic groups of order 6: $\mathbb{Z}_6$ and S_3.

$\langle \mathbb{Z}_6, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]



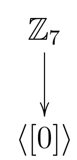
$\langle S_3, \circ \rangle$	(1)	(123)	(132)	(12)	(13)	(23)
(1)	(1)	(123)	(132)	(12)	(13)	(23)
(123)	(123)	(132)	(1)	(13)	(23)	(12)
(132)	(132)	(1)	(123)	(23)	(12)	(13)
(12)	(12)	(23)	(13)	(1)	(132)	(123)
(13)	(13)	(12)	(23)	(123)	(1)	(132)
(23)	(23)	(13)	(12)	(132)	(123)	(1)



$\mathbb{Z}_6$ is a cyclic group, with generators [1] and [5] that is also a permutation group. $\mathbb{Z}_6$ is also a group of symmetries since $\mathbb{Z}_6 \cong D_3$. S_3 is strictly a permutation group. It is neither cyclic or abelian.

There is exactly one group of order 7 up to isomorphism.

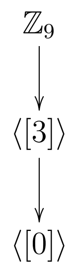
$\langle \mathbb{Z}_7, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[0]	[1]	[2]	[3]	[4]	[5]



We can see that $\mathbb{Z}_7$ is cyclic where every nonidentity element is a generator. This group is a permutation group.

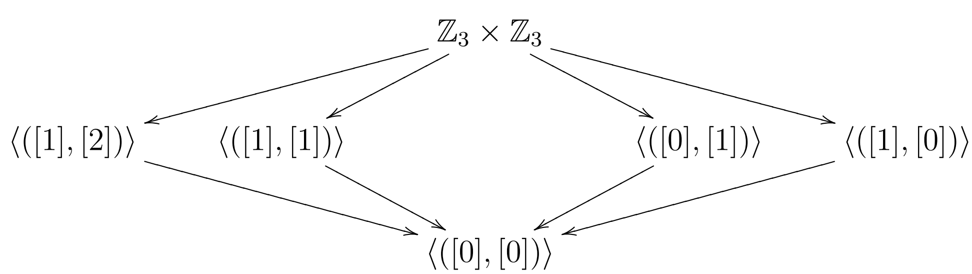
There are two groups of order 9 up to isomorphism.

$\langle \mathbb{Z}_9, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]



$\mathbb{Z}_9$ is a cyclic, permutation group with generators $[1], [2], [4], [5], [7]$, and $[8]$.

$\langle \mathbb{Z}_3 \times \mathbb{Z}_3, \oplus \rangle$	([0], [0])	([0], [1])	([0], [2])	([1], [0])	([1], [1])	([1], [2])	([2], [0])	([2], [1])	([2], [2])
([0], [0])	([0], [0])	([0], [1])	([0], [2])	([1], [0])	([1], [1])	([1], [2])	([2], [0])	([2], [1])	([2], [2])
([0], [1])	([0], [1])	([0], [2])	([0], [0])	([1], [1])	([1], [2])	([1], [0])	([2], [1])	([2], [2])	([2], [0])
([0], [2])	([0], [2])	([0], [0])	([0], [1])	([1], [3])	([1], [0])	([1], [1])	([2], [2])	([2], [0])	([2], [1])
([1], [0])	([1], [0])	([1], [1])	([1], [2])	([2], [0])	([2], [1])	([2], [2])	([0], [0])	([0], [1])	([0], [2])
([1], [1])	([1], [1])	([1], [2])	([1], [0])	([2], [1])	([2], [2])	([2], [0])	([0], [1])	([0], [2])	([0], [0])
([1], [2])	([1], [2])	([1], [0])	([1], [1])	([2], [2])	([2], [0])	([2], [1])	([0], [2])	([0], [0])	([0], [1])
([2], [0])	([2], [0])	([2], [1])	([2], [2])	([0], [0])	([0], [1])	([0], [2])	([1], [0])	([1], [1])	([1], [2])
([2], [1])	([2], [1])	([2], [2])	([2], [0])	([0], [1])	([0], [2])	([0], [0])	([1], [1])	([1], [2])	([1], [0])
([2], [2])	([2], [2])	([2], [0])	([2], [1])	([0], [2])	([0], [0])	([0], [1])	([1], [2])	([1], [0])	([1], [1])



$\mathbb{Z}_3 \times \mathbb{Z}_3$ is abelian but not cyclic. It is used in number theory.

There is one group each for orders 11 and 13 respectively.

$\langle \mathbb{Z}_{11}, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[9]	[10]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[9]	[10]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[9]	[10]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[9]	[10]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[9]	[9]	[10]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[10]	[10]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

$$\mathbb{Z}_{11}$$
$$\downarrow$$
$$\langle [0] \rangle$$

We can see that $\mathbb{Z}_{11}$ is cyclic where every nonidentity element is a generator. This group is a permutation group.

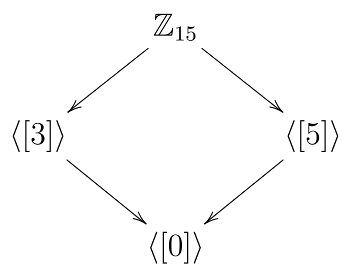
$\langle \mathbb{Z}_{13}, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[9]	[9]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[10]	[10]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
[11]	[11]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[12]	[12]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

$$\mathbb{Z}_{13}$$
$$\downarrow$$
$$\langle [0] \rangle$$

We can see that $\mathbb{Z}_{13}$ is cyclic because every $a \in \mathbb{Z}_{13}$ such that $a \neq [0]$ is a generator. This group is a permutation group.

There is one group of order 15 up to isomorphism: $\mathbb{Z}_{15}$.

$\langle \mathbb{Z}_{15}, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[9]	[9]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[10]	[10]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
[11]	[11]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[12]	[12]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
[13]	[13]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
[14]	[14]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]



$\mathbb{Z}_{15}$ is cyclic, permutation group with generators $[1], [2], [4], [7], [8], [11], [13], [14]$.

This is the sole group, up to isomorphism, of order 17.

$\langle \mathbb{Z}_{17}, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[9]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[10]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
[11]	[11]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[12]	[12]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
[13]	[13]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
[14]	[14]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
[15]	[15]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
[16]	[16]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

$$\mathbb{Z}_{17}$$
$$\downarrow$$
$$\langle [0] \rangle$$

Because this group is cyclic of prime order, every nonidentity element is a generator. This group is a permutation group.

This is the only group of order 19 up to isomorphism. Since $n = 19$ is a prime number, and $\mathbb{Z}_{19}$ is cyclic, $\mathbb{Z}_{19} = \langle [a] \rangle$ for all $[a] \in \mathbb{Z}_{19}$, $a \neq [0]$. This group is a permutation group.

$\langle \mathbb{Z}_{19}, \oplus \rangle$	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]
[7]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[8]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
[9]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[10]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
[11]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
[12]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
[13]	[13]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
[14]	[14]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
[15]	[15]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
[16]	[16]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
[17]	[17]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
[18]	[18]	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]

$$\mathbb{Z}_{19}$$
$$\downarrow$$
$$\langle [0] \rangle$$

Bibliography

[1] Eric W. Weisstein. 'Finite Group.' From *MathWorld* – A Wolfram Web Resource. http://mathworld.wolfram.com/FiniteGroup.html

[2] Durbin, John R. 5th ed. Hoboken, NJ: John Wiley & Sons, Inc., 2005.