MTH 243 LEARNING OUTCOMES

Learning outcomes for MTH243 Multivariable Calculus.

Introduction to functions of two and more variables:
Goals. The goal of this chapter is to see that many quantitites in various scientific fields depend on more than one varible: the strength of the gravitational force between two bodies depend on their masses and their distance apart; the monthly mortgage payments depend on the amount borrowed, the interest rate, and the number of years to pay off. Then we will see many different ways of representing functions of several variables including algebraic formulas, graphs, contour diagrams, cross sections, and numerical tables.

Contents.

1. functions of two variables given by: contour diagram, table, formula 3-space: cartesian coordinates, graphing points, planes, distance, equation of a sphere, cylinders
2. graphs of functions of two variables, cross-sections: algebraic description of cross-sections (vibrating string); contour diagrams: graphing contour diagrams, finding contour diagrams algebraically and from tables; maple tools;
3. linear functions: equation, properties, graph, contour diagram, table; finding an equation of a linear function (a plane) given by 3 points in the 3-space
4. functions of three variables $f(x, y, z)$: representing a function of three variables via level surfaces (= level sets); quadratic surfaces;
5. limits and continuity; given two continuous functions their sum, difference, product, composition, and quotient (unless we divide by zero) is a continuous function

Vector calculus in dimension two and three:
Goals. The primary goal of this chapter is to familiarize the students with the notion of vectors as representing quantities that have directions as well as magnitude. For example velocity of a moving object in space is given by a vector to specify how fast and in what direction it is moving. Also we will study two important operations involving vectors, scalar product and cross product, and their applications to linear geometry in space including equations of planes and the volume of paralellopiped.

Contents.

1. Displacement vectors: addition, subtraction, and scalar multiplication, magnitude. Operations with vectors: addition, substaction, scalar multiplication. Parallel vectors. Coordinate vectors $\vec{i}$, $\vec{j}$, $\vec{k}$. Components of a vector. Position vectors.

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2. Vectors are used for describing quantities having direction and magnitude. Some examples of vectors from physics: velocity, acceleration, force. Properties of vector operatons. Vectors in n dimensions.
3. The dot product of vectors: algebraic and geometric definition; algebraic properties of the dot product. Orthogonal (perpendicular) vectors.
4. Applications of dot product: Normal vectors of planes. Geometry of lines and planes in three dimensions. Resolving a vector into components via projections. (Interpretation of dot product as work).
5. The cross product: algebraic and geometric definition. The right hand rule. Algebraic properties of the cross product.
6. Computing determinants. The area of a parallelogram and a triangle. Applications to linear geometry in 3D.

Differentiation:
Goals. The goal is to understand how the value of a multivariable function changes as one of its independent variables is allowed to vary with all other variables fixed at constants. Hence we will study the rate of change of a multivariable function with respect to each of its independent variables, introducing the notion of *partial derivatives*. We will then use these partial derivetive to get various local information about the function including tangent planes and directional derivatives. Furthermore we will develop various techniques such as the second derivative tests and Lagrange multiplier methods to find local and global maxima and minima of a multivariable function.

Contents.

1. Partial derivatives: definition, notation, difference quotients. Computing partial derivatives algebraically. Approximation of partial derivatives using contour diagrams. Visualizing partial derivatives on a graph. Interpreting partial derivatives using units.
2. Local linearity, an approximation by a linear function; the tangent plane.
   The differential.
3. The gradient vector. The directional derivative. The relation of the gradient vector and directional derivatives at a point. Geometric properties of the gradient vector. Gradient vector in cartesian coordinates. Gradient vector of a function of three variables and its properties. Tangent planes to level surfaces.
4. The chain rule for functions of several variables.
5. Second order partial derivatives. Equality of Mixed partial derivatives.
6. Taylor approximations: linear and quadratic approximations.
7. Local extrema, critical points. The second derivative test.

Lagrange multipliers, the lagrangian function. Local minima and maxima under inequality constraints.

Integration:

Goals. The goal is to define the double and triple integrals as a limit of Riemann sum and to see their interpretations as average value, volume under graph, volume of a solid, area of a region, total mass from density. We will evaluate these integrals using iterated integral. Furthermore we will set up double integrals in polar coordinates and triple integrals in cylindrical and spherical coordinates.

Contents.

1. The definite integral of a function of two variables. Riemann sums and the definition of the double integral. Area of a region. Volume. Average value of a function over a region.
2. Computing double integrals using iterated integrals. Reversing the order of integration.
3. Triple integrals: definition via Riemann sums. Iterated integrals. Volume.
4. Polar coordinates, integration in polar coordinates. Cylindrical coordinates, integration in cylindrical coordinates. Spherical coordinates, integration in spherical coordinates.
5. Parametric description of curves in 2D and 3D: circle, ellipse, line, graph of $f(x)$. Vector valued functions. Parametric equations of a curve in vector notation.
6. Applications: velocity, speed, tangent lines. The length of a curve.
7. Vector fields. A flow line (= an integral line) of a vector field. The flow. Gradient vector fields.
8. Parametrized surfaces. Important examples: cylinders, graphs of a function of two variables $z = f(x, y)$, planes, spheres (use spherical coordinates), surfaces of revolution.
9. Line integrals and their evaluation. Computing line integrals over parametrized curves. Interpretation of the line integral as work over a curve and as circulation of a vector fields around a closed curve.
10. Gradient fields and path-independent fields. The fundamental theorem for line integrals. A potential function. Path-independent vector fields, gradient vector fields.
11. The curl.
    Green's theorem.
12. The flow and flux of a vector field. Orientation of a surface and the area vector. The flux integral. Computing flux integrals through surfaces which are graphs of functions, cylinders, and spheres.

13. The divergence of a vector field. The divergence in cartesian coordinates. Divergence free vector fields.
    The divergence theorem.