MTH141 Section 003 Optional Practice Exam

Name: (2 points extra credit each)

Problem 1(3.1;59) For what values of a and b is the line

$$2x + y = b$$

tangent to the parabola $y = ax^2$ when $x = 2$?

Ans: $a = -1/2$, $b = 2$

Problem 2(3.1;31) If f and g are the functions whose graphs are shown in problem 31 on page 205 in the text. let $u(x) = f(x) \cdot g(x)$ and $v(x) = f(x)/g(x)$. Find $u'(1)$ and $v'(5)$.

Ans: $u'(1) = 0$, $v'(5) = -2/3$

Problem 3(3.4;13) Prove that

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

You may assume the formulas for the derivatives of sin and cos. ans: apply the quotient rule

Problem 4(3.5;67) A curve C is defined by the parametric equations

$$x = t^2, \quad y = t^3 - 3t$$

Show that C has two tangents at the point $(3, 0)$ and find their equations. Ans:

$$y = \sqrt{3}x + 3\sqrt{3}, \quad y = -\sqrt{3}x + 3\sqrt{3}$$

Problem 5(3.7;27) Find the derivative of

$$y = \sin^{-1} x^2$$

Simplify where possible. Ans:

$$y' = \frac{2x}{\sqrt{1 - x^4}}$$

Problem 6(3.7;33) Find the derivative of

$$y = x^{\sin x}$$

Ans:

$$y' = x^{\sin x} \left(\frac{\sin x}{x} + \ln x \cos x \right)$$

Problem 7(4.1;25) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and the volume V satisfy the equation

$$PV = C$$

where C is a constant. Suppose that at a certain instant the volume is $600cm^3$, the pressure is $150kPa$, and the pressure is increasing at the rate of $20kPa/min$. At what rate is the volume decreasing at this instant?

Ans: $80cm^3/min$

Problem 8(4.1;3) If

$$y = x^3 + 2x \quad \text{and} \quad \frac{dx}{dt} = 5$$

find dy/dy when $x = 2$.

Ans: 70

Problem 9(3.8;5) Find the linear approximation of the function

$$f(x) = \sqrt{1-x}$$

at $a = 0$ and use it to approximate $\sqrt{0.9}$ and $\sqrt{0.99}$.

Ans:

$$\sqrt{1-x} = f(x) \approx f(0) + f'(0)(x-0) = 1 - \frac{1}{2}x$$

so

$$\sqrt{0.9} \approx 1 - \frac{1}{2}(0.1) = 0.95$$

and

$$\sqrt{0.99} \approx 1 - \frac{1}{2}(0.01) = 0.995$$

Problem 10(3.7;38) Find y' if

$$x^y = y^x$$

Ans:

$$y' = \frac{\ln y - y/x}{\ln x - x/y}$$

Problem 11(3.7;17) Differentiate

$$y = \ln\left(e^{-x} + xe^{-x}\right)$$

Ans:

$$y' = -\frac{x}{1+x}$$