UNIVERSITY OF RHODE ISLAND

Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

Location: Lippitt Hall 204

Time: Monday, April 30, 2018, 1:00pm

(refreshments at 12:50 p.m.)

No Formula, no Problem: Companion Matrices and the Polynomial Rootfinding Problem

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Abstract: Let $p(\lambda)$ be a *monic scalar polynomial of degree* n such that

$$p(\lambda) = \lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-2}\lambda^2 + a_{n-1}\lambda + a_n\,.$$

The matrix C_p is said to be a *companion matrix* of $p(\lambda)$ if $\det(\lambda I - C_p) = \alpha p(\lambda)$ where α is a nonzero scalar. An example of a companion matrix is

$$C_p = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & \ddots & \ddots & \\ & & & 1 & 0 \end{bmatrix}_{n\times n}.$$

Since $\det(\lambda I - C_p) = p(\lambda)$, the eigenvalues of C_p are exactly the roots of $p(\lambda)$.

In this talk, we show how to construct many different companion matrices. We then discuss how one can find roots of monic and non-monic polynomials using this matrix. For *non-monic* polynomials we introduce the notion of *companion pencils*, which are just algebraic expressions of the form $A - \lambda B$ where A and B are square matrices. Finally, we discuss how to transform square matrices and matrix pencils to Schur form and generalized Schur form, respectively. When these results are applied to computing eigenvalues of companion matrices and pencils, we then in turn get roots of scalar polynomials, monic or non-monic.

Everyone is encouraged to attend, in particular undergraduate and graduate students.