MTH 535 Fall 2004 Study guide for final exam

1. Know the definitions of the following terms: *ring, σ-ring, field, σ-field, coutably additive, subadditive, measure, measure space, probability space, product σ-field, x-slice, product measure.* Also $\limsup$ and $\liminf$ applied both to sequences of functions and sequences of sets (and relationship to "i.o." and "a.a").

2. Be able to describe the σ-fields $\mathcal{M}, \mathcal{B}$ of Lebesgue measurable and Borel measurable sets on $\mathbf{R^n}$.

3. Be able to *outline* the construction of $\mathcal{M}$, in particular the definition and simple properties of Lebesgue outer measure.

4. Know how to define what it means to say that f is measurable w.r.t. a σ-field $\mathcal{F}$. Be able to define *upper semicontinous at x.*

5. For measurable f, be able to define $\int f\,d\mu$ starting from the case where f is *simple*, then *non-negative* , then in general in terms of f_-, f_+. Also be able to define the Riemann integral for a function f on a finite interval.

6. Be able to state the following: Borel-Cantelli Lemmas, Chebyshev Inequality, Monotone Convergence Theorem, Dominated Convergence Theorem, Fatou's Lemma, Fubini's Theorem (all forms)

7. Be able to define *convergence almost everywhere, convergence in mean, convergence in measure*, know which of these implies others, and under what circumstances, and give examples of sequences converging in some of these senses but not others.

8. Know the definition and simple properties of the Cantor set.

9. Know some examples of measures other than Lebesgue measure, in particular, counting measure and how it can be used to think of infinite series as integrals, and measures of the form $\mu = \sum c_k \delta_x$ or $\nu(A) = \int_A f\,d\mu$.

10. Be able to describe the probability interpretation of σ-fields, measureable functions, and integrals, as events,random variables, and expectations. Know the definition of the *probability distribution (measure)* of a random variable, and the statement of the Laws of Large numbers discussed in class. Be able to define *independent r.v.'s.*

11. Know examples of the following: an uncountable set of Lebesgue measure 0, a function that is Lebesgue integrable but not Riemann integrable. a seqnence of functions that converges in measure but not a.e., a function f on $\mathbf{R}$ such that f is integrable but f^2 is not, and another such that f^2 is integrable but f is not,

a sequence of functions for which there is strict inequality in Fatou's lemma.

12. Be able to give a proof of the first Borel-Cantelli lemma, Chebyshev's inequality, the fact that if f is both Riemann and Lebesgue integrable then the integrals are equal, Fatou's lemma from the Monotone Convergence Theorem, and the following theorems from the text:

(I'll list these later–there won't be too many.)