Some answers / solutions

7.18 $Y \sim Exp(\tfrac{1}{2}) = \Gamma(1, \tfrac{1}{2})$

7.20 a) $g(y) = \begin{cases} \frac{3y^2}{2} + \frac{3(-y)^2}{2} & \text{for } 0 < y < 1 \\ 0 & \text{else} \end{cases}$

b) Using $Z = Y^2 = u(Y)$ with $u(y) = y^2$ and inverse $w(z) = \sqrt{z}$ we get $h(z) = g(w(z))\, w'(z)$

$0 \le y \le 1$

$0 \le z \le 1$

$$= \begin{cases} \frac{6(\sqrt{z})^2}{2} \cdot \frac{1}{2} z^{-1/2} = \frac{3}{2}\sqrt{z}, & 0 \le z \le 1 \\ 0 & \text{else} \end{cases}$$

7.36 Let $\begin{cases} y_1 = x_1^2 \\ y_2 = x_1 x_2 \end{cases}$. Solve for x_1, x_2: $\begin{cases} x_1 = y_1^{1/2} \\ x_2 = y_2 / y_1^{1/2} = y_2 y_1^{-1/2} \end{cases}$

The support of $f(x_1, x_2)$ is the set $\{(x_1, x_2) \mid 0 \le x_1, x_2 \le 1\}$:

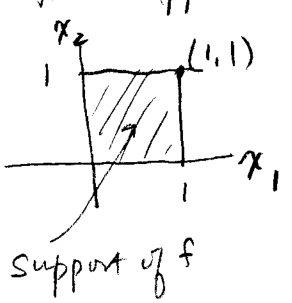

Fig. 1

The corresponding (y_1, y_2) lie in the set $\{(y_1, y_2) \mid 0 \le y_1 \le 1,\ 0 \le y_2^2 \le y_1\}$ shown in Fig. 2

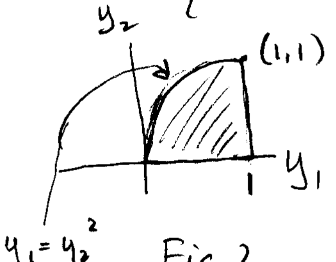

Fig. 2

One way to see this is to note that $0 \le x_2 \le 1 \Rightarrow y_2^2 = x_1^2 x_2^2 \le x_1^2 = y_1$.

Another way is to look at where the sides of the square in Fig. 1 are mapped into Fig. 2, e.g. the side $(0,0) \to (0,1)$ is mapped to the point $(0,0)$, the side $(1,0) \to (1,1)$ is mapped to the curve $y_1 = y_2^2$, etc

Then $\begin{vmatrix} \partial x_1/\partial y_1 & \partial x_1/\partial y_2 \\ \partial x_2/\partial y_1 & \partial x_2/\partial y_2 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} y_1^{-1/2} & 0 \\ \cdot & y_1^{-1/2} \end{vmatrix} = \frac{1}{2} \cdot \frac{1}{y_1}$ so the joint density of Y_1, Y_2 is

$$g(y_1, y_2) = \begin{cases} 4 y_1^{1/2} y_2 y_1^{-1/2} \cdot \frac{1}{2} \frac{1}{y_1} = 2 y_2 / y_1 & \text{if } \begin{matrix} 0 \le y_1 \le 1 \\ 0 \le y_2^2 \le y_1 \end{matrix} \\ 0 & \text{else} \end{cases}$$