

Relative efficiency as a ratio of sample sizes

Suppose that $\hat{\Theta}_1^{(n)}$ and $\hat{\Theta}_2^{(n)}$ are two (unbiased) estimators of the parameter $\hat{\Theta}$ based on X_1, X_2, \dots, X_n , a sample of size n . We call the ratio

$$\frac{\text{Var}(\hat{\Theta}_1^{(n)})}{\text{Var}(\hat{\Theta}_2^{(n)})} \quad (1)$$

the *relative efficiency* of $\hat{\Theta}_2^{(n)}$ with respect to $\hat{\Theta}_1^{(n)}$. Remember that we consider $\hat{\Theta}_1^{(n)}$ better than $\hat{\Theta}_2^{(n)}$ if this ratio is < 1 . It often happens that $\text{Var}(\hat{\Theta}_1^{(n)}) = V_1/n$ and $\text{Var}(\hat{\Theta}_2^{(n)}) = V_2/n$ for two constants V_1, V_2 that don't depend on n , for example, if each X_i has variance σ^2 and $\hat{\Theta}^{(n)}$ is the sample mean \bar{X} then $\text{Var}(\hat{\Theta}^{(n)}) = \sigma^2/n$. Then for two sample sizes n_1 and n_2 , we will have

$$\text{Var}(\hat{\Theta}_1^{(n_1)}) = \text{Var}(\hat{\Theta}_2^{(n_2)})$$

when $V_1/n_1 = V_2/n_2$, that is, when $V_1/V_2 = n_1/n_2$. But V_1/V_2 equals the relative efficiency ratio in (1) so we can interpret this as follows if we use the variance to assess accuracy:

The ratio of sample sizes needed to get the same accuracy with the two estimators is given by the relative efficiency. If this ratio is expressed as a percentage, then that percentage of the sample size needed for a given accuracy with estimator 2 is the sample size needed for the same accuracy with estimator 1.