MTH 452 4/10/08 Name (Print): Key

1. Suppose $X_1, X_2, \ldots, X_7 \overset{i.i.d.}{\sim} Exp(\theta)$ and we want to test the null hypothesis

$$H_0 : \theta = 5$$

against the alternative

$$H_1 : \theta = 2$$

Let $X = X_1 + X_2 + \cdots + X_7$.

a) Use the N-P lemma to show that a test which rejects H_0 if $X \leq K$ is a most powerful test of its size against. H_1

a) Explain how to determine the value K so that the test which rejects H_0 when $X \leq K$ has size .01.

a) $L_0 = \prod_{i=1}^{7} \frac{1}{5} e^{-x_i/5} = \left(\frac{1}{5}\right)^7 e^{-\sum_1^7 x_i/5}$

$L_1 = \prod_{i=1}^{7} \frac{1}{2} e^{-x_i/2} = \left(\frac{1}{2}\right)^7 e^{-\sum_1^7 x_i/2}$

NP implies that a most powerful test of its size rejects H_0 if $\frac{L_0}{L_1} \leq k$ for some k. But $\frac{L_0}{L_1} \leq k \Leftrightarrow \left(\frac{2}{5}\right)^7 e^{-\sum_1^7 x_i\left(\frac{1}{5}-\frac{1}{2}\right)} \leq k$

$\Leftrightarrow \ln\left(\frac{L_0}{L_1}\right) \leq \ln k \Leftrightarrow 7\ln(.4) - \left(\sum_1^7 x_i\right)(-.3) \leq \ln k$

$\Leftrightarrow \sum_1^7 x_i \leq \frac{\ln k - 7\ln(.4)}{.3} = K$

i.e. $X \leq K$

b) $X \sim \Gamma(7,5)$ under H_0

So we choose K so that $P(0 \leq X \leq K) = .01$ when $X \sim \Gamma(7,5)$

(Use software or tables of Γ probabilities)