HW Solutions

14.11 $$Var\left(\frac{X}{\sigma_1}+\frac{Y}{\sigma_2}\right) = Var\left(\frac{X}{\sigma_1}\right) + Var\left(\frac{Y}{\sigma_2}\right) + \frac{2\,cov(X,Y)}{\sigma_1\sigma_2}$$

$$= 1 + 1 + 2\rho = 2+2\rho$$

$$2+2\rho \geq 0 \Rightarrow 1+\rho \geq 0 \Rightarrow \rho \geq -1$$

Similarly $Var\left(\frac{X}{\sigma_1}+\frac{Y}{\sigma_2}\right) = 2-2\rho$ and

$$2-2\rho \geq 0 \Rightarrow 1-\rho \geq 0 \Rightarrow \rho \leq 1.$$

$$\therefore \quad -1 \leq \rho \leq 1$$

14.13 let $q = \sum_1^n [y_i - \beta x_i]^2$. We find β which minimizes q!

$$\frac{dq}{d\beta} = 2\sum_1^n [y_i - \beta x_i]\cdot(-x_i) = 0 \Rightarrow -\sum x_i y_i + \beta \sum x_i^2 = 0$$

$$\Rightarrow \beta = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$\frac{d^2 q}{d\beta^2} = 2\sum x_i^2 \geq 0$$ So this is a minimum.

14.17 $$\sum [y_i - (\hat{\alpha} + \hat{\beta} x_i)]^2 = \sum [y_i - \bar{y} + \hat{\beta}\bar{x} - \hat{\beta} x_i]^2$$

using $\alpha = \bar{y} - \hat{\beta}\bar{x}$.

But then $$\sum [y_i - \bar{y} + \hat{\beta}\bar{x} - \beta x_i]^2 = \sum \left[(y_i - \bar{y}) - \hat{\beta}(x_i - \bar{x})\right]^2$$

$$= \sum (y_i - \bar{y})^2 - 2\hat{\beta}\sum (x_i - \bar{x})(y_i - \bar{y}) + \hat{\beta}^2 \sum (x_i - \bar{x})^2$$

$$= S_{yy} - 2\hat{\beta} S_{xy} + \hat{\beta}^2 S_{xx} \quad \left(\text{Use } \hat{\beta} = \frac{S_{xy}}{S_{xx}}\right)$$

$$= S_{yy} - \frac{2 S_{xy}^2}{S_{xx}} + \frac{S_{xy}^2}{S_{xx}^2} S_{xx}$$

$$= S_{yy} - \frac{S_{xy}^2}{S_{xx}} = S_{yy} - \hat{\beta} S_{xy}.$$