12.7 Under H_0, $\bar{X} = \frac{1}{2}(X_1+X_2) \sim N(\mu_0, \frac{1}{2})$ since $Var(\bar{x}) = \frac{1}{2}$. Size α = prob of rejecting H_0 when H_0 is true so $\alpha = P(\bar{X} > \mu_0+1) = P(\bar{x}-\mu_0 > 1)$

$$= P\left(\frac{\bar{X}-\mu_0}{\sqrt{1/2}} > \frac{1}{\sqrt{1/2}}\right) = P(Z > \sqrt{2}) = .079$$

(since $\frac{\bar{X}-\mu_0}{\sqrt{1/2}} = Z \sim N(0,1)$)

12.10 Proceed as in Example 12.4 except that $\mu_0 - \mu_1 > 0$ here so the inequality for the critical region is reversed.

12.11 - done in class via Maple.

12.15 $L_0 = \frac{1}{(2\pi)^{n/2}\sigma_0^n} e^{-\frac{1}{2\sigma_0^2}\sum x_i^2}$, $L_1 = \frac{1}{(2\pi)^{n/2}\sigma_1^n} e^{-\frac{1}{2\sigma_1^2}\sum x_i^2}$

so $\frac{L_0}{L_1} = \left(\frac{\sigma_1}{\sigma_0}\right)^n e^{-\frac{\sum x_i^2}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)}$

$\sigma_1 > \sigma_0 \Rightarrow \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} > 0$ so $\frac{L_0}{L_1} \le k$

is equivalent to $\sum x_i^2 \ge K$ for some K.

Since e^{-tc} is a decreasing function of t when $c > 0$ $\left(c = \frac{1}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)\right)$,

Equivalently, we reject if $\sum_1^n \left(\frac{X_i}{\sigma_0}\right)^2 \ge K'$

and since $\frac{X_i}{\sigma_0} \sim N(0,1)$ under $H_0: \sigma = \sigma_0$ we have $\sum_1^n \left(\frac{X_i}{\sigma_0}\right)^2 \sim \chi^2(n)$. So the most powerful test of size α rejects H_0 if $\sum \left(\frac{X_i}{\sigma_0}\right)^2 \ge \chi_n^2$ (book answer is X??)