

## MTH 452 Study Guide for Exam I

1. You should be able to give the *definitions* of the following terms:

a) random sample, i.i.d., unbiased estimate, minimum variance unbiased estimate, asymptotically unbiased estimator, sufficient statistic, consistent estimator, maximum likelihood estimator, method of moments estimator,

b) sample mean, sample variance, moment generating function, order statistics of a sample, likelihood function,  $(1 - \alpha)$ -level confidence interval for a parameter.

c) Relative efficiency of one unbiased estimate with respect to another and how this is related to sample sizes needed for equivalent variance.

2. You should bring the table of distributions and their properties. You should know the relations between chi-square distributions and sums of squares of i.i.d. normal r.v.'s, the relation between sums of i.i.d exponential r.v.s and gamma r.v.s, how the chi-square and t-densities arise when estimating parameters from a normal population. You should know how to use the Central Limit Theorem to find normal approximations, and how to look up  $z_\alpha$  and  $t_\alpha$  values in a table that will be provided, as well as define  $z_\alpha$  and  $t_\alpha$ .

3. You should be able to state and use the following *theorems* and PROVE the ones indicated with a \*.

a) The Law of Large Numbers, The Central Limit Theorem, Theorems 7.1, 7.3, 8.4\*, 8.10\*, 8.13\*, 8.11, 10.2, 10.4, 11.1\*, 11.2\*, 11.3, 11.4, the

$$\sum_1^2 (X_i - \mu)^2 = \sum_1^n (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2 \text{ identity*}.$$

4. a) Be able to derive the density and/or distribution function of a function of a r.v. whose density or d.f. is given using the d.f. technique or Theorem 7.1, and find the moments of these transformed r.v.s.

b) Be able to derive the density of the first and last order statistics (min and max) for a sample  $X_1, \dots, X_n$  given the distribution of each  $X_i$ .

c) Be able to deduce distribution properties of sums of i.i.d. random variables using m.g.f.s, e.g. that the sum of two i.i.d exponential r.v.s is a certain

gamma r.v.

5. Be able to

- a) Show that an estimate is unbiased or asymptotically unbiased.
- b) Show that an estimator is sufficient using the factorization theorem
- c) Show that an unbiased estimator has minimum variance using the Cramer-Rao inequality.
- d) Find a method of moments estimator.
- e) Find a maximum likelihood estimator.
- f) Find a confidence interval for the mean of a normal sample.

6. Be able to do problems similar to ones in homework and quizzes.

Note: If a topic is not on this list I won't ask about it, e.g. transformation of two r.v.s, densities of order statistics other than  $Y_1$  and  $Y_n$ , etc.