MTH 142 Practice Problems for Exam 2 — Last updated 10/19/04

Sections 7.5, 7.6, 7.7, 7.8, 8.1, 8.2, 8.3, 8.4, 8.6, 8.7

1. Use numerical methods to approximate the given integral. Do the calculations by hand, write down details. (Section 7.5)

   a) Use midpoint rule with $n = 3$ to approximate $\int_0^4 \frac{1}{1+x}\, dx$.

   b) Use the trapezoid rule with $n = 4$ to approximate $\int_{-2}^0 -x^2\, dx$

2. The following table gives values of a function f, whose concavity does not change in the interval [0,2]:

x	0.0	.25	.50	.75	1.00	1.25	1.50	1.75	2.00
f(x)	0.0	1.492	2.08	2.48	2.75	2.92	3.00	2.98	2.86

   We want to estimate $\int_0^2 f(x)dx$. (Sections 7.5, 7.6)

   a) Find the midpoint estimate with 4 subdivisions, MID(4).

   b) You can easily calculate that LEFT(4) = 3.915 and RIGHT(4) = 5.345. Find the trapezoid and Simpson's estimates TRAP(4) and SIMP(4).

3. The numerical approximation of an integral $\int_a^b f(x)dx$ with LEFT(5),RIGHT(5),TRAP(5), and MID(5), produced the numbers shown below (not necessarily in the same order):

   $$3.6725, \quad 3.6745, \quad 3.6762, \quad 3.6800$$

   It is known that the function $f(x)$ is decreasing and concave up. (Section 7.5, 7.6)

   a) Choose a suitable method for each number. Give a mathematical justification.

   b) If the exact value of the integral $\int_a^b f(x)dx$ is 3.6757 what is the error in each case.

   c) How many decimals do you predict will be correct for each of the methods LEFT, RIGHT, MID and TRAP if $n = 50$ is used to approximate the integral? Explain.

4. Say which of the following integrals are improper. For those that are improper, determine if they are convergent or divergent. Do not use the comparison test.

   (a.) $\int_1^\infty \frac{3}{\sqrt{2+x}}\, dx$ (b.) $\int_{-1}^5 \frac{1}{2x+2}\, dx$ (c.) $\int_0^5 \frac{2}{t^2+3t}\, dt$ (d.) $\int_{-\infty}^\infty e^{3t}\, dt$

5. Use the Comparison Test to determine which of the following improper integrals converge.

   (e) $\int_1^\infty \frac{x}{\sqrt{1+x^6}}\, dx$ (f) $\int_2^\infty \frac{t^2+1}{t^2-1}\, dt$

6. A solid S is produced by revolving about the x-axis the region R in the plane bounded by $y = 1$, $y = 2x^2$, and $x = 0$. Use the method of sections to find the exact volume of the resulting solid. (Sections 10.1, 10.2)

7. Answer the question in the previous problem, only now the solid is produced by revolving R about the y-axis. (Sections 10.1, 10.2)

8. Answer the question in the previous problem, only now the solid is produced by revolving R about the line $x = -1$. (Sections 10.1, 10.2)

9. An oil slick has the shape of a circle with radius 7,000 feet. After measurements were taken, it has been determined that the density of the oil at "r" feet from the center of the circle is given by the formula $d(r) = \frac{0.004}{1+r^2}$ Kg/ft^2. Use the method of sections to express the mass of the oil as an integral. (Section 8.3)

10. A 1 m. long rod has (linear) density $d(x) = 2.0 + 0.015x$ gr/m, where x is the distance from one end. Calculate the total mass and the center of mass of the rod. (Section 8.3)

11. A metallic plate in the first quadrant is limited by the lines $x = 0$, $y = 0$ and the curve $y = (3 - x)/(1 + x)$, where x is in cm. Its density is 0.15 gr/cm^2. Find the total mass of the plate, and find the center of mass $(\bar{x}, \bar{y})$. (Section 8.3)

12. A cylindrical container has as base a circle with radius 2 ft. and has height 3 ft. The container is filled with water containing a kind of bacteria that seems to accumulate near the bottom of the container, where more food is found. A researcher took samples and determined that the density of bacteria is a function of the distance y to the bottom in inches, as follows: $d = 3.5 - 0.05\,y$ million bacteria per cubic inch
Determine the total number of bacteria in the container. (Section 8.3)

13. A 20 feet tall water tank has the shape of an inverted cone (i.e, the vertex is at the bottom) with circular top with radius 10 feet and height 20 feet. Water weighs 62.4 lb/ft^3.

    a) Use the method of sections to obtain a sum that approximates the work required to take the water out of the tank from the top.

    b) obtain the exact work in part (a) by calculating a suitable integral. Calculate the integral by hand -show your work. (Section 8.4)

14. A dam has the shape of a trapezoid, with horizontal parallel sides measuring 30 ft. (bottom) and 60 ft (top). The height of the dam is 30 ft., and one vertical side is perpendicular to both base and top. The dam has water up to the top on one side.

    a) Use the method of sections to obtain a sum that approximates the total force excerted by the water on the dam. Recall that water weighs 62.4 lb/ft^3.

    b) Obtain an integral that gives the total force of the water on the dam. (Section 8.4)

15. The lenght x (inches) of nails in a shipment has distribution with density function $p(x) = 2/x^2$, where x is a number between 1 and 2 (so no nails have length less than 1 inch, or more than 2 inches).
    a) What fraction of the shipment consists of nails with length between 1.5 and 1.7 inches?
    b) What fraction of the shipment consists of nails with length more than 1.5 inches? (Section 8.6)

16. For each density function calculate the median T, and the mean $\bar{x}$ (Section 8.7) :
    (a) $p(x) = 2/x^2$, where $1 \le x \le 2$;   (b) $p(x) = 2e^{-2x}$, where $0 \le x < \infty$.

17. For the given density function find the cumulative distribution function $P(x)$ (Section 8.7):
    (a) $p(x) = 2/x^2$, where $1 \le x \le 2$;   (b) $p(x) = 2e^{-2x}$, where $0 \le x < \infty$.

18. Problem 15 in page 388 of the text. (Section 8.6)