Maple Project 2 MTH 142
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PROBLEM 1.

a) It can be shown mathematically that

$$S = \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

Verify that Maple produces the same result.

b) Define the the partial sum function

$$partialsum(N) = \sum_{n=1}^{N} \frac{1}{n^4}, \quad N = 1, 2, \ldots$$

Make sure the your Maple function gives output in decimal notation. Produce a pointplot of $partialsum(N)$, for $N = 1, 2, \ldots, 20$. Does this plot suggest that the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges? Explain.

c) The error when approximating S by $partialsum(N)$ is defined as

$$Err(N) = S - partialsum(N)$$

Calculate $Err(1)$, $Err(10)$, $Err(100)$. What do these numbers suggest about the rate at which the error goes to zero as N is increased?

PROBLEM 2.

a) Define in Maple the function $f(x) = \dfrac{1}{2 + 0.8\sin(x)}$, and obtain the Taylor polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$ of $f(x)$ for x near $a = 3.2$.

b) Plot $f(x)$ together with $P_1(x)$, $P_2(x)$, $P_3(x)$, for $0 \le x \le 6$. Insert a comment identifying the function and the polynomials by their colors in the plot.

c) The error when approximating f(x) by $P_n(x)$ at x is defined as

$$E_n(x) = f(x) - P_n(x) = \text{exact value - approximation}$$

Calculate $E_1(3.0)$, $E_2(3.0)$, $E_3(3.0)$, and $E_4(3.0)$. Comment on the numbers you obtain.

d) Produce a plot of $f(x)$ and $P_{20}(x)$ with $0 \le x \le 6$ and $0 \le y \le 2$. What is your guess for the interval of convergence of the Taylor series of $f(x)$ for x near $a = 3.2$?

COMMENTS and additional information

- Your name, date, class and section should be at the top of your paper. The final project should have only one author. You may discuss the project with your classmates, but what you turn in should contain your own original answers. Plagiarism is considered a serious offence.

- Before each Maple computation, you should insert an explanation of what you are about to do. Neatness and good English will be taken into account.

- Maple should be *used in all calculations and plots.*

- For additional information on Plotting, solving equations and calculating integrals in maple: see the maple worksheet *" Introduction to Maple in Calculus II"(intro142.mws)* , located in `www.math.uri.edu` → `Online Teaching Materials` → `Maple Worksheets for Calculus II` → `Introduction to Maple in Calculus II`

USEFUL MAPLE COMMANDS

```
> restart;                  # good to have this at the top of worksheet;
> with(student);           # adds extra functionality
> with(plots);             # adds extra functionality for plots
> f:=x->x^2;               # define a function
> f:=x->evalf(x^2);        # define a function, force it to give decimal result
> plot([f(x),g(x)],x=0..5);# plot two functions
> p1:=pointplot({seq([n,g(n)],n=1..10)}):
                           # plot points (1,g(1)), (2,g(2)),...,(10,g(10)),
                           # and store the plot under the name p1
                           # NOTE: when a plot is stored under a name, it won't show.
                           # In this case, to see the plot use ''display''
> display([p1,p2]);        # show together two plots called p1 and p2
> Pi                       # the number 3.1415... Note the capital P.
> taypol:=n->convert( taylor(f(x),x=1,n+1),polynom);
                           # taypol(5) gives the Taylor polynomial of degree 5 of
  # the function f(x) about x = 1;
> sub(x=2,p);              # substitute x=2 into p
> sum(1/n,n=1..infinity);  # infinite sum from n=1 to infinity of 1/n
```