Math 142 Review for Exam 1

1. $\int \frac{6t+1}{t^3+2t^2+t}\,dt = \int \frac{6t+1}{t(t+1)^2}\,dt$ $\qquad \frac{6t+1}{t(t+1)^2} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$

$6t+1 = A(t+1)^2 + Bt(t+1) + Ct$

$= At^2 + 2At + A + Bt^2 + Bt + Ct$

$A+B=0$

$2A+B+C=6$

$A=1$

$B=-1$

$C=5$

$= \int \left(\frac{1}{t} - \frac{1}{t+1} + \frac{5}{(t+1)^2}\right) dt$

$= \boxed{\ln|t| - \ln|t+1| - \frac{5}{t+1} + C}$

2. $\int \ln(x+1)\,dx = \int \ln w\,dw = w\ln|w| - \int dw$

$w = x+1$ $\quad u = \ln w$ $\quad du = \frac{1}{w}dw$

$dw = dx$ $\quad dv = dw$ $\quad v = w$

$= w\ln|w| - w + C$

$= \boxed{(x+1)\ln|x+1| - (x+1) + C}$

3. $\int_0^1 (-3x+2)e^{2x}\,dx = \frac{(-3x+2)}{2}e^{2x}\Big|_0^1 + \frac{3}{2}\int_0^1 e^{2x}\,dx$

$u = -3x+2$ $\quad du = -3\,dx$

$dv = e^{2x}dx$ $\quad v = \frac{1}{2}e^{2x}$

$= \left[-\frac{3x+2}{2}e^{2x} + \frac{3}{4}e^{2x}\right]\Big|_0^1$

$= -\frac{1}{2}e^2 + \frac{3}{4}e^2 - \left(e^0 + \frac{3}{4}e^0\right)$

$= \boxed{\frac{1}{4}e^2 - \frac{7}{4} \approx .0973}$

4. $\int_{-\pi/4}^{0} \cos x\sqrt{3\sin x+4}\,dx = \frac{1}{3}\int_{x=-\frac{\pi}{4}}^{x=0} \sqrt{w}\,dw = \frac{1}{3}\left(\frac{2}{3}w^{3/2}\right)\Big|_{x=-\pi/4}^{x=0}$

$w = 3\sin x + 4$

$\frac{1}{3}dw = \cos x\,dx$

$= \frac{2}{9}(3\sin x+4)^{3/2}\Big|_{-\pi/4}^{0}$

$= \boxed{\frac{2}{9}\left[4^{3/2} - \left(-\frac{3\sqrt{2}}{2}+4\right)^{3/2}\right] \approx 1.206}$

5. $\int_0^1 \frac{x^2\,dx}{2x-3} = \int_0^1 \left[\frac{1}{2}x + \frac{3}{4} + \frac{9/4}{2x-3}\right] dx$

$$\begin{array}{r} \frac{1}{2}x + \frac{3}{4} \\ 2x-3\,\overline{)\,x^2+0x+0} \\ -(x^2 - \frac{3}{2}x) \\ \hline \frac{3}{2}x \\ -(\frac{3}{2}x - \frac{9}{4}) \\ \hline \frac{9}{4} \end{array}$$

$\frac{x^2}{4} + \frac{3}{4}x + \frac{9}{8}\ln|2x-3|\Big|_0^1$

$= \frac{1}{4} + \frac{3}{4} + \frac{9}{8}\ln 1 - \left(0 + 0 + \frac{9}{8}\ln 3\right)$

$= \boxed{1 - \frac{9}{8}\ln 3 \approx -.2359}$

6. $\int \frac{x+4}{x^2+8x-9}\,dx = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln|w|+C = \boxed{\frac{1}{2}\ln|x^2+8x-9|+C}$

$w = x^2+8x-9$
$dw = 2x+8\,dx$

7. $\int \frac{-2}{x^2+x+\frac{26}{4}}\,dx = \int \frac{-2\,dx}{(x+\frac{1}{2})^2+(\frac{5}{2})^2} = -2\int \frac{5/2\,d\theta}{(5/2)^2\tan^2\theta+(5/2)^2}\cdot\frac{1}{\cos^2\theta}$

$x^2+x+\frac{1}{4}-\frac{1}{4}+\frac{26}{4}$

$x+\frac{1}{2} = \frac{5}{2}\tan\theta$
$dx = \frac{5}{2}\,\frac{d\theta}{\cos^2\theta}$

$= -\frac{4}{5}\int \frac{d\theta}{(\tan^2\theta+1)\cos^2\theta}$

$= -\frac{4}{5}\int d\theta$

$= -\frac{4}{5}\theta + C = \boxed{-\frac{4}{5}\arctan\left[\frac{2}{5}x+\frac{1}{5}\right]}$

8. $\int \frac{e^x}{e^x+2}\,dx = \int \frac{dw}{w} = \ln|w|+C = \boxed{\ln|e^x+2|+C}$

$w = e^x+2$
$dw = e^x dx$

9. $\int_1^3 \frac{t^2}{t^2+9}\,dt = \int_{t=1}^{t=3} \frac{9\tan^2\theta}{9\tan^2\theta+9}\,\frac{3}{\cos^2\theta}\,d\theta$

$t = 3\tan\theta$
$dt = 3\,\frac{1}{\cos^2\theta}\,d\theta$

$= 3\int_{t=1}^{t=3} \tan^2\theta\,d\theta = 3\int_{t=1}^{t=3}\left(\frac{1}{\cos^2\theta}-1\right)d\theta$

$= 3\left[\tan\theta-\theta\right]_{t=1}^{t=3}$

$= 3\left[\frac{t}{3}-\arctan\frac{t}{3}\,\Big|_1^3\right]$

$= \boxed{3\left[1-\arctan 1-\left(\frac{1}{3}-\arctan\frac{1}{3}\right)\right]}$

$\boxed{\approx .6091}$

10. $\int \frac{dx}{1+\sqrt{x}} = \int \frac{2(w-1)\,dw}{w}$

$w = 1+\sqrt{x}$
$dw = \frac{1}{2\sqrt{x}}\,dx$
$dx = 2\sqrt{x}\,dw$
$= 2(w-1)\,dw$

$= 2\int\left(1-\frac{1}{w}\right)dw$

$= 2\left[w-\ln|w|\right]+C$

$= \boxed{2\left[1+\sqrt{x}-\ln|1+\sqrt{x}|\right]+C}$

11. $\int 4t\sqrt{2t+1}\,dt = \int 2(w-1)\sqrt{w}\,dw$

$w = 2t+1$
$dw = 2dt$
$\frac{1}{2}dw = dt$

$t = \frac{w-1}{2}$

$= 2\int\left(w^{3/2}-w^{1/2}\right)dw$

$= \frac{4}{5}w^{5/2}-\frac{4}{3}w^{3/2}+C$

$= \boxed{\frac{4}{5}(2t+1)^{5/2}-\frac{4}{3}(2t+1)^{3/2}+C}$

12. $\int a\sin(bt)\,dt = \boxed{-\frac{a}{b}\cos(bt)+C}$

13. $\int \frac{3x^2}{\sqrt{16-x^2}}\,dx$ $\quad x = 4\sin\theta \quad dx = 4\cos\theta\,d\theta \quad \frac{x}{4}=\sin\theta \quad \theta = \arcsin\frac{x}{4} \quad \cos\theta = \frac{1}{4}\sqrt{16-x^2}$

$$= \int \frac{3\cdot 16\sin^2\theta\cdot 4\cos\theta\,d\theta}{\sqrt{16-16\sin^2\theta}} = 48\int \sin^2\theta\,d\theta$$

$\int \sin^2\theta\,d\theta = -\sin\theta\cos\theta + \int\cos^2\theta\,d\theta$

$u = \sin\theta \quad du = \cos\theta\,d\theta$

$dv = \sin\theta\,d\theta \quad v = -\cos\theta\,d\theta$

$\int\sin^2\theta\,d\theta = -\sin\theta\cos\theta + \theta - \int\sin^2\theta\,d\theta$

$\int\sin^2\theta\,d\theta = -\frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2} + C$

$= -\frac{1}{2}\left(\frac{x}{4}\right)\left(\frac{1}{4}\sqrt{16-x^2}\right) + \frac{1}{2}\arcsin\frac{x}{4}$

$\therefore \int\frac{3x^2}{\sqrt{16-x^2}}dx = 48\left[-\frac{1}{2}\left(\frac{x}{4}\right)\frac{1}{4}\sqrt{16-x^2} + \frac{1}{2}\arcsin\frac{x}{4}\right] + C$

$= \boxed{-\frac{3}{2}x\sqrt{16-x^2} + 24\arcsin\frac{x}{4} + C}$

14. $\int\sqrt{3-x^2}\,dx$ $\quad x = \sqrt{3}\sin\theta \quad dx = \sqrt{3}\cos\theta\,d\theta \quad \sin\theta = \frac{x}{\sqrt{3}} \quad \theta = \arcsin\frac{x}{\sqrt{3}} \quad \cos\theta = \frac{1}{\sqrt{3}}\sqrt{3-x^2}$

$$= \int\sqrt{3-3\sin^2\theta}\,\sqrt{3}\cos\theta\,d\theta = 3\int\cos^2\theta\,d\theta$$

$\int\cos^2\theta\,d\theta = \sin\theta\cos\theta + \int\sin^2\theta\,d\theta$

$u = \cos\theta \quad du = -\sin\theta\,d\theta$

$dv = \cos\theta\,d\theta \quad v = \sin\theta$

$\int\cos^2\theta\,d\theta = \frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} + C$ (same trick as #13)

$= \frac{1}{2}\,\frac{x}{\sqrt{3}}\,\frac{1}{\sqrt{3}}\sqrt{3-x^2} + \frac{1}{2}\arcsin\frac{x}{\sqrt{3}} + C$

$\therefore \int\sqrt{3-x^2}\,dx = \boxed{\frac{1}{2}x\sqrt{3-x^2} + \frac{3}{2}\arcsin\frac{x}{\sqrt{3}} + C}$

15. $\int\frac{-6x+2}{3x^2-2x}\,dx = -\int\frac{dw}{w} = -\ln|w| + C$

$= \boxed{-\ln|3x^2-2x| + C}$

$w = 3x^2-2x$

$dw = (6x-2)\,dx$

$-dw = (-6x+2)\,dx$

16. $\int \frac{4dx}{6x-x^2}$ $\qquad \frac{4}{x(6-x)} = \frac{A}{x} + \frac{B}{6-x}$ $\qquad 4 = 6A - Ax + Bx$

$-A+B=0$

$6A=4$

$A=\frac{2}{3}$

$B=\frac{2}{3}$

$= \int \left(\frac{2/3}{x} + \frac{2/3}{6-x}\right)dx$

$= \boxed{\frac{2}{3}\ln|x| - \frac{2}{3}\ln|6-x| + C}$

17. $\int \frac{t-4}{t^3+8t^2}dt = \int \frac{(t-4)\,dt}{t^2(t+8)}$ $\qquad \frac{t-4}{t^2(t+8)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+8}$

$t-4 = At(t+8) + B(t+8) + Ct^2$

$t-4 = At^2 + 8At + Bt + 8B + Ct^2$

$A+C=0 \qquad C=-3/16$

$8A+B=1 \qquad A=3/16$

$8B=-4 \qquad B=-1/2$

$= \int \left(\frac{3/16}{t} - \frac{1/2}{t^2} - \frac{3/16}{t+8}\right) + C$

$= \boxed{\frac{3}{16}\ln|t| + \frac{1}{2t} - \frac{3}{16}\ln|t+8| + C}$

18. $\int \frac{3t-2}{t^3+4t}dt = \int \frac{(3t-2)dt}{t(t^2+4)}$ $\qquad \frac{3t-2}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4}$

$3t-2 = At^2 + 4A + Bt^2 + Ct$

$A+B=0$

$C=3 \qquad B=1/2$

$4A=-2$

$A=-1/2$

$= \int \left(\frac{-1/2}{t} + \frac{\frac{1}{2}t+3}{t^2+4}\right)dt$

$= -\frac{1}{2}\ln|t| + \frac{1}{2}\int \frac{t}{t^2+4}dt + 3\int \frac{dt}{t^2+4}$

$w = t^2+4 \qquad t = 2\tan\theta$

$dw = 2t\,dt \qquad dt = 2\frac{1}{\cos^2\theta}d\theta$

$= -\frac{1}{2}\ln|t| + \frac{1}{4}\int \frac{dw}{w} + 3\int \frac{2\,d\theta}{(4\tan^2\theta+4)\cos^2\theta}$

$= -\frac{1}{2}\ln|t| + \frac{1}{4}\ln|t^2+4| + \frac{3}{2}\theta + C$

$= \boxed{-\frac{1}{2}\ln|t| + \frac{1}{4}\ln|t^2+4| + \frac{3}{2}\arctan\frac{t}{2} + C}$

19. $\int \frac{(\sqrt{x}+1)^2}{\sqrt{x}} = 2\int w^2dw = \frac{2}{3}w^3 + C$

$= \boxed{\frac{2}{3}(\sqrt{x}+1)^3 + C}$

$w = \sqrt{x}+1$

$dw = \frac{1}{2\sqrt{x}}dx$

$2dw = \frac{1}{\sqrt{x}}dx$

20. $\int \frac{t+t^7}{\sqrt{t}}\,dt = \int (\sqrt{t} + t^{13/2})\,dt = \boxed{\frac{2}{3}t^{3/2} + \frac{2}{15}t^{15/2} + C}$

21. $\int \frac{5}{x^3-16x}\,dx = \int \frac{5\,dx}{x(x-4)(x+4)}$ $\qquad \frac{5}{x(x+4)(x-4)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+4}$

$5 = Ax^2 - 16A + Bx^2 + 4Bx + Cx^2 - 4C$

$A+B+C=0 \qquad B=C$

$4B-4C=0 \qquad -\frac{5}{16}+2B=0$

$-16A=5$

$A=\frac{-5}{16} \qquad B=C=\frac{5}{32}$

$= \int \left(\frac{-5/16}{x} + \frac{5/32}{x-4} + \frac{5/32}{x+4} \right) dx$

$= \boxed{-\frac{5}{16}\ln|x| + \frac{5}{32}\ln|x-4| + \frac{5}{32}\ln|x+4| + C}$

22. $\int_0^{\pi/4} \cos^5(2t)\,dt = \frac{1}{2}\int_{t=0}^{t=\pi/4} \cos^5 w\,dw$

$w=2t$

$\frac{1}{2}dw = dt$

$= \frac{1}{2}\left(-\frac{1}{5}\cos^4 w\sin w + \frac{4}{5}\int \cos^3 w\,dw\right)$

$= -\frac{1}{10}\cos^4 w \sin w + \frac{2}{5}\left(-\frac{1}{3}\cos^2 w\sin w + \frac{2}{3}\int \cos w\,dw\right)$

$= -\frac{1}{10}\cos^4(2t)\sin(2t) - \frac{2}{15}\cos^2(2t)\sin(2t) + \frac{4}{15}\sin(2$

$= \boxed{\frac{4}{15}}$

23. $\int \frac{\pi^4}{\sin^4 x}\,dx = \pi^4\left[-\frac{1}{3}\frac{\cos x}{\sin^3 x} + \frac{2}{3}\int \frac{1}{\sin^2 x}dx\right] + C$

$= \pi^4\left[-\frac{1}{3}\frac{\cos x}{\sin^3 x} + \frac{2}{3}\left(\frac{-1}{1}\frac{\cos x}{\sin x}\right)\right] + C$

$= \boxed{\pi^4\left(-\frac{1}{3}\frac{\cos x}{\sin^3 x} - \frac{2}{3}\frac{\cos x}{\sin x}\right) + C}$

24. $\int_0^{2\pi} \sin^2(mx)\,dx = \frac{1}{m}\int_{x=0}^{x=2\pi} \sin^2 w\,dw$

$w = mx$

$dw = m\,dx$

$= \frac{1}{m}\left[-\frac{1}{2}\sin x\cos x + \frac{1}{2}\int dw\right]\Big|_{x=0}^{x=2\pi}$

$= \frac{1}{m}\left[-\frac{1}{2}\sin x\cos x + \frac{1}{2}w\Big|_{x=0}^{x=2\pi}\right]$

$= \frac{1}{m}\left[-\frac{1}{2}\sin(mx)\cos(mx) + \frac{1}{2}mx\Big|_0^{2\pi}\right]$

$= \frac{1}{m}(\pi m) = \boxed{\pi}$

25. $\int t^2 e^{t-1}\,dt = \frac{1}{e}\int t^2 e^t\,dt$

III-14

$= \boxed{\frac{1}{e}\left[t^2e^t - 2te^t + 2e^t\right] + C}$

26. $\int (2x+1)^2 \sin x\,dx = \boxed{-(2x+1)^2\cos x + 4(2x+1)\sin x + 8\cos x + C}$

III-15

27. $\int x^8 \ln(x^5)\,dx = 5\int x^8 \ln x\,dx$

III-13

$= 5\left[\frac{1}{9}x^9\ln x - \frac{1}{81}x^9\right] + C$

$= \boxed{\frac{5}{9}x^9\ln x - \frac{5}{81}x^9 + C}$