Maple Project 1 MTH 142 Fall 2004

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PROBLEM 1. *(About of one of the versions of the Fundamental Theorem of Calculus.)*

1.1 Define in Maple the functions

$$g(t) = e^{\frac{1}{2}-\sqrt{t}} \quad \text{and} \quad G(x) = \int_0^x g(t)dt$$

What is G(1) ? (in decimal form).

1.2 Define in Maple

$$h(t) = \frac{G(t+0.1) - G(t)}{0.1}$$

Plot $h(t)$ and $g(t)$ (not $G(t)$) on the same set of coordinate axes for $0 \leq t \leq 2$. Insert a comment and explanation about similarities of the two curves.

PROBLEM 2. *(About the behavior of the error when calculating integrals numerically.)*

2.1 Use Maple to define the function $f(t) = 4\sqrt{1-t^2}$. Then verify that $\int_0^1 f(t)dt = \pi$.

2.2 Use Maple to define the "error" functions (where n is a positive integer)

$$E_{right}(n) = RIGHT(n) - \pi \quad \text{and} \quad E_{mid}(n) = MID(n) - \pi$$

Here $RIGHT(n)$ and $MID(n)$ are the right Riemann sum and the midpoint approximations to $\int_0^1 f(t)dt$. Calculate $E_{right}(10)$ and $E_{mid}(10)$ in decimal form.
Note: in maple, "midpoint" is called "middlesum".

2.3 Plot $E_{right}(n)$ and $E_{mid}(n)$ versus n in the same set of coordinate axes. Use $2 \leq n \leq 30$ (n is an integer). (Use the pointplot command) Explain the plot.

PROBLEM 3. *(About improper integrals.)*

3.1 Define in Maple the function

$$r(t) = \frac{\sin(t)}{t}\, dt$$

Plot $r(t)$ for $1 \leq t \leq 40$.

3.2 Define in Maple the function

$$R(b) = \int_1^b \frac{\sin(t)}{t}\, dt$$

From the graph in 3.1, do you expect $R(b)$ to be an (increasing/decreasing/none) function of b? Why?

3.3 Plot $R(b)$ for $1 \leq b \leq M$, for some large M. Make a prediction about the convergence or divergence of the integral

$$\int_1^\infty r(t)dt$$

Explain.

COMMENTS and additional information

- Your name, date, class and section should be at the top of your paper. The final project should have only one author. You may discuss the project with your classmates, but what you turn in should contain your own original answers. Plagiarism is considered a serious offense.
- Before each Maple computation, you should insert a comment with an explanation of what you are about to do. Try to be clear in your explanation.
- Maple should be *used in all calculations and plots.*
- For additional information on Plotting, solving equations and calculating integrals in maple: see the maple worksheet *" Introduction to Maple in Calculus II"(intro142.mws)* , located in `www.math.uri.edu/Center/workc2.html`
- MAPLE HELP will be available in some of the labs. The schedule and location will be announced at the web site `www.math.uri.edu`

USEFUL MAPLE COMMANDS

Insert (in text mode "**T**") your name, class, date at the top of your worksheet.
Then, in input mode ">" type the following lines before you type the answers to questions:
(Note: to supress output, use ":" instead of ";" to finish input line)

```
> restart;                  # good to have this at the top of worksheet;
> with(student);            # adds leftsum, rightsum, trapezoid functions (and others)
> with(plots);              # adds extra functionality for plots
```

Remember to precede each Maple computation with a explanatory comment in text mode.
Here are some useful commands:

```
> f:=x->x^2;                # define a function
> f:=x->evalf(x^2);         # define a function, force it to give decimal result
> evalf(%)                  # give previous output in decimal form
> plot(f(x),x=0..5);        # plot a function f(x)
> plot(f(x),x=0..5,y=0..3); # plot a function f(x) with a particular window
> plot([f(x),g(x)],x=0..5);# plot two functions on same system of coordinate axes
> leftsum(f(t),t=0..1,6);  # left Riemann sum of f(t) with 6 subintervals
> p1:=pointplot({seq([n,g(n)],n=1..10)}):
                            # plot points (1,g(1)), (2,g(2)),...,(10,g(10)),
                            # and store the plot under the name p1
> display([p1,p2]);         # show two plots called p1 and p2 in one set of coordinate axes
> Pi                        # the number 3.1415...It must have a capital P.
```

Also, when you define a function in terms of an integral in Maple, make sure the integration variable is not used again in other calculations. Sometimes this creates some problems.