MTH 244 - Additional Problems for §1.6

Section 1 (Merino) and section 3 (Dobrushkin) - February 2003

Using substitutions in differential equations.

1. In each exercise, solve the given differential equation of the form $y' = F(ax+by+c)$ where $b \neq 0$ by using transformation $v = ax + by + c$ (therefore $v' = a + by'$).

   (a) $y' = e^{x+y-1} - 1$,
   (b) $y' = \sin(x+y)$,
   (c) $yy' + xy^2 = x$,
   (d) $y' = (2x + y - 1)^2 - 1$,
   (e) $y' = e^{x+y}/(x+y) + 1$,
   (f) $y' = \sqrt{4x+y}$.

   Hint: $\int \frac{dv}{1+\sin v} = \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + C$.

2. In each exercise, solve the given differential equation by using transformation $v = xy$.

   (a) $y' = y^2$,
   (b) $x\,y' = y/(xy+1)$,
   (c) $x\,y' = y\,e^{xy} - y$,
   (d) $x^2\,y' = \cos^2(xy) - xy$.

Homogeneous Equations.

1. In each exercise, determine whether or not the function is homogeneous. If it is homogeneous, state the degree of the function.

   (a) $5x^2 - 2xy + 3y^2$,
   (b) $\sqrt{x^2 + xy - y^2}$,
   (c) $y + \sqrt{x^2 + 2y^2}$,
   (d) $x\,\cos(y/x) - y\,\sin(x/y)$,
   (e) $\frac{x^2+2xy}{x+y}$
   (f) e^x
   (g) $\sqrt{x-y}$,
   (h) $x^3 - xy + y^3$,
   (i) $(x^2+y^2)\,\exp\left(\frac{2x}{y}\right)$,
   (j) $\ln|x| - \ln|y|$,
   (k) $\tan\frac{x}{2y}$,
   (l) $(x^2+y^2)^{3/2}$.

2. In each exercise, determine whether or not the equation is with a homogeneous right-hand side function. If it is, find the general solution of the equation.

   (a) $y' = \frac{y}{x} + 2\sqrt{\frac{x}{y}}$,
   (b) $2xy\,y' = x^2 + y^2$,
   (c) $x^2y' = y^2 + 2xy - 6x^2$,
   (d) $y' = e^{y/x} + \frac{y}{x}$,
   (e) $3xy\,dx + 4x^2\,dy = 0$,
   (f) $(y^3 - yx^2)\,dx + 2xy^3\,dy = 0$,
   (g) $(x^2+y^2)\,dx + xy\,dy = 0$,
   (h) $2x\,dx + (3y^2 + x^2 + xy)\,dy = 0$,
   (i) $x^3\,y' + 3xy^2 + 2y^3 = 0$,
   (j) $y' = \frac{2x+y}{x}$,
   (k) $x^2y' = 2y^2 + xy$,
   (l) $x^2y' = y^2 + 3xy + x^2$,
   (m) $\left(\frac{y}{x}+1\right)\,dx + \left(\frac{x}{y}+1\right)\,dy = 0$,
   (n) $(x^4+y^4)\,dx - 2xy^3\,dy = 0$,
   (o) $y^2\,dx + (x^2+xy+y^2)\,dy = 0$,
   (p) $y(y/x-1)\,dx + x(x/y+1)\,dy = 0$,
   (q) $(2y+xy)\,dx + (2xy+y^2)\,dy = 0$,
   (r)

3. Solve the given differential equation with a homogeneous right-hand side function. Then determine the value of the constant needed for solving the initial value problem.

   (a) $(y+\sqrt{x^2+y^2})\,dx - x\,dy = 0, \quad y(\sqrt{3}) = 1,$
   (b) $(x-y)\,dx + (3x+y)\,dy = 0, \quad y(3) = -2,$
   (c) $y(3x+2y)\,dx - x^2\,dy = 0, \quad y(1) = 2,$
   (d) $xy\,dx + 2(x^2+2y^2)\,dy = 0, \quad y(0) = 1,$
   (e) $(3x^2-2y^2)\,y' = 2xy, \quad y(0) = 1,$
   (f) $(x^3+y^3)\,dx + 3x^2y\,dy = 0, \quad y(1) = 1.$

4. Solve the following equations with linear coefficients.

   (a) $(3x+y-4)\,dx + (x-y+1)\,dy = 0,$
   (b) $(4x+2y-8)\,dx + (2x-y)\,dy = 0,$
   (c) $(x-4y-9)\,dx + (4x+y-2)\,dy = 0,$
   (d) $(x+y+1)\,dx + (y-x-3)\,dy = 0,$
   (e) $(x+2y+2)\,dx + (2x+3y+2)\,dy = 0,$
   (f) $(3x+2y+4)\,dx - (x-y+6)\,dy = 0,$
   (g) $(x-4y+1)\,dx + (x+2y+1)\,dy = 0,$
   (h) $(x+y-2)\,dx + (x+1)\,dy = 0,$
   (i) $(5y-10)\,dx + (2x+4)\,dy = 0,$
   (j) $(x-1)\,dx - (3x-2y-5)\,dy = 0,$
   (k) $(2x-y-5)\,dx - (x-2y-1)\,dy = 0,$
   (l) $(2x+y-2)\,dx - (x-y+4)\,dy = 0,$
   (m) $(2x-y+5)\,dx + (4x+y+1)\,dy = 0,$
   (n) $(3x+y+1)\,dx + (x+3y+11)\,dy = 0.$

5. Find the indicated particular solution for each of the following equations.

   (a) $(11x-9y-2)\,y' + (4x+11y+15) = 0, \quad y(0) = 1,$
   (b) $(y+2)\,dx - (x+y+2)\,dy = 0, \quad y(1) = 1,$
   (c) $(x+y+1)\,dx + (y-x-3)\,dy = 0, \quad y(1) = 1,$
   (d) $(x-y+1)\,dx = (x+y)\,dy, \quad y(0) = 0,$
   (e) $(x+y+1)\,dx + (y-x-1)\,dy = 0, \quad y(1) = 2,$
   (f) $(x+1)\,dx = (x+2y+2)\,dy, \quad y(0) = 2,$
   (g) $(x - y\,e^{y/x} + y\,e^{-y/x})\,dx = x(e^{-y/x} + e^{y/x})\,dy, \; y(1) = 0,$
   (h) $y\,e^{x/y}\,dx + (y - x\,e^{x/y})\,dy = 0, \quad y(0) = 1.$

6. In order to get some practice in using these techniques when the variables are designated as something other than x and y, solve the following equations.

   (a) $\frac{dw}{dv} = \frac{w+v-4}{3v+3w-8},$
   (b) $\frac{dp}{dq} = \frac{q-4p+5}{3q+12p+2},$
   (c) $\frac{dk}{ds} = \frac{3k+2s-1}{k-4s+2},$
   (d) $\frac{dx}{dt} = \frac{x+3t+3}{t-2},$
   (e) $\frac{dr}{d\theta} = \frac{2r-\theta+1}{r+\theta+7},$
   (f) $\frac{dw}{dt} = \frac{t-w+5}{t-w+4},$
   (g) $\frac{dz}{du} = \frac{u-3z}{z+u},$
   (h) $\frac{dt}{dx} = \frac{2x-t-5}{x-5t+11}.$

7. Solve the following equations.

   (a) $\frac{dy}{dx} = \left(\frac{x-y+1}{2x-2y}\right)^2,$
   (b) $\frac{dy}{dx} = \left(\frac{x-y++2}{x+1}\right)^2,$

8. Solve the equation $y' = y^p/x^q$, where p and q are positive integers.