MTH 244 - Additional Problems for §1.4

Section 1 (Merino) and section 3 (Dobrushkin) - January 2003

1. Find the general solution of the following linear differential equations with constant coefficients (Recall that $\sinh x = 0.5\,e^{x} - 0.5\,e^{-x}$ and $\cosh x = 0.5\,e^{x} + 0.5\,e^{-x}$).

   a) $y' + 4y = 17\,\sin x$,
   b) $y' + 4y = e^{-4x}$,
   c) $y' - 2y = 2 + 4x$,
   d) $y' - 2y = e^{2x}$,
   e) $y' + 2y = 4$,
   f) $y' + 2y = e^{-2x}$,
   g) $y' + 2y = 3\,\sinh x$,
   h) $y' - y = 4\,\cosh x$,
   i) $y' + 4y = 2\,e^{-2x}$,
   j) $y' - 2y = 4$,
   k) $y' - 2y = 3\,e^{-x}$,
   l) $y' - 2y = 5\,\sin x$,
   m) $y' + 2y = 4\,e^{2x}$,
   n) $y' + 2y = 3\,\cosh x$,
   o) $y' - y = 4\,\sinh x$,
   p) $y' = 2y + x^2 + 3$.

2. Find the general solution of the following linear differential equations with variable coefficients

   a) $y' + xy = x$;
   b) $x\,y' + (3x + 1)\,y = e^{-3x}$;
   c) $x^2 + xy = 1$;
   d) $x\,y' + (2x + 1)\,y = 4x$.

3. Find the general solution of the given differential equation

   a) $x\,y' = y + x^2\,e^{x}$;
   b) $y' = (y - 1)\,\tan x$;
   c) $y' + 2x\,y = 4x$;
   d) $(1 + x)\,y' = xy + x^2$.

4. Solve the given initial value problems.

   a) $y' + 2y = 10, \quad y(0) = 8$;
   b) $y' = y + 6x^2, \quad y(0) = -2$;
   c) $x^2y' + 2xy - x + 1 = 0, \quad y(1) = 0$;
   d) $x\,y' = y + 2x^2, \quad y(5) = 1$.

5. Find a continuous solution of the following Cauchy problems:

   $$(\mathbf{a})\ \ y' + 2y = f(x), \quad y(0) = 0; \qquad (\mathbf{b})\ \ y' + y = f(x), \quad y(0) = 0,$$

   where

   $$f(x) = \begin{cases} 1, & 0 \le x \le 3; \\ 0, & x > 1. \end{cases}$$

6. Which nonhomogeneous linear ordinary differential equations of first order are separable?

7. One of the main contaminants of nuclear accident at Chernobyl is strontium-90, which decays at a constant rate of approximately 2.47% per year. What percent of the original strontium-90 would still remain after 100 years?

8. In an RL-series circuit it is given that $L = 1 + t^2$ henries, $R = -t$ ohms, $V(t) = t$, and $I(0) = 1$ amperes. Compute the value of the current at any time.

9. In an RL-series circuit it is given that $L = t$ henries, $R = 2t + 1$ ohms, $V(t) = 4t$, and $I(1) = 2$ amperes. Compute the value of the current at any time.

10. Find charge in a simple RC-series circuit with electro-motive force $E(t) = t$ volts. It is given that $R = t$ ohms, $C = (1 + t)^{-1}$ farads, and $Q(1) = 1$ coulomb.

Short Answers to Some Problems

1.

(1a) $y = C\,e^{-4x} + 4\sin x - \cos x$
(1b) $y = C\,e^{-4x} + x\,e^{-4x}$
(1c) $y = C\,e^{2x} - 2 - 2x$
(1d) $y = C\,e^{2x} + x\,e^{2x}$
(1e) $y = C\,e^{-2x} + 2$
(1f) $y = C\,e^{-2x} + x\,e^{-2x}$
(1g) $y = C\,e^{-2x} - 2\sinh x - \cosh x$
(1h) $y = C\,e^{x} + 2x\,e^{x} - e^{-x}$
(1i) $y = C\,e^{-4x} + e^{-2x}$
(1j) $y = C\,e^{2x} - 2$
(1k) $y = C\,e^{2x} - e^{-x}$
(1l) $y = C\,e^{2x} - e^{-2x}\,(2\sin x + \cos x)$
(1m) $y = C\,e^{-2x} + e^{2x}$
(1n) $y = C\,e^{-2x} - \sinh x + 2\cosh x$
(1o) $y = C\,e^{x} + 2x\,e^{x} + e^{-x}$
(1p) $y = Ce^{2x} - \frac{1}{4}(7 + 2x + 2x^2)$

2.

a) $\mu(x) = e^{-x^2/2}$, $y = 1 + c\,e^{-x^2/2}$
b) $\mu(x) = e^{3x}$, $\quad y(x) = e^{-3x} + \frac{C}{x}\,e^{-3x}$
c) $\mu(x) = x^{-1}$, $\quad xy = \ln|x| + C$
d) $y = \frac{C}{x}\,e^{-2x} + 2 - \frac{1}{x}$

3.

a) $y = Cx + x(x-1)\,e^{x}$
b) $y(x) = 1 + C/\cos x$
c) $y(x) = (2\,x^2 + C)\,e^{-x^2}$
d)$y = \frac{1}{1+x}(Ce^{x} - 2 - 2x - x^2)$

4.

a) $y = 5 + C\,e^{-2x}$, $C = 3$
b) $y = 10\,2^{x} - 6(x^2 + 2x + 2)$
c) $y = \frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}$
d) $y = 2x^2 + \frac{49}{5}x$