Additional Problems for Section 1.3

"VARIABLES SEPARABLE"

PROBLEMS

Solve each of the equations in Problems 1 through 7.

1. $\dfrac{dy}{dx} = \dfrac{x^2}{y}$

2. $\dfrac{dy}{dx} = \dfrac{x^2}{y(1 + x^3)}$

3. $\dfrac{dy}{dx} + y^2 \sin x = 0$

4. $\dfrac{dy}{dx} = 1 + x + y^2 + xy^2$

5. $\dfrac{dy}{dx} = (\cos^2 x)(\cos^2 2y)$

6. $x\dfrac{dy}{dx} = (1 - y^2)^{1/2}$

7. $\dfrac{dy}{dx} = \dfrac{x - e^{-x}}{y + e^{y}}$

Find the solution of each of the equations in Problems 8 through 12 that satisfies the given initial condition.

8. $\sin 2x\, dx + \cos 3y\, dy = 0; \qquad y(\pi/2) = \pi/3$

•9. $x\, dx + ye^{-x}\, dy = 0, \qquad y(0) = 1$

10. $\dfrac{dr}{d\theta} = r, \qquad r(0) = 2$

11. $\dfrac{dy}{dx} = \dfrac{\ln |x|}{1 + y^2}, \qquad y(1) = 0$

12. $\dfrac{dy}{dx} = xy^3(1 + x^2)^{-1/2}, \qquad y(0) = 1$

13. Solve the equation

$$y^2(1 - x^2)^{1/2}\, dy = \sin^{-1} x\, dx$$

in the interval $-1 < x < 1$.

14. Solve the equation

$$\frac{dy}{dx} = \frac{ax + b}{cx + d},$$

where a, b, c, and d are constants.

15. Solve the equation

$$\frac{dy}{dx} = \frac{ay + b}{cy + d},$$

where a, b, c, and d are constants.

*16. Show that the equation

$$\frac{dy}{dx} = \frac{y - 4x}{x - y}$$

is not separable, but that if the variable y is replaced by a new variable v defined by $v = y/x$, then the equation is separable in x and v. Find the solution of the given equation by this technique.