Maple Project 2 MTH 243 Fall 2009

The purpose of this homework is to use Maple to treat optimization problems from graphical, numerical and algebraic points of view.

PART 1. Consider the function

$$f(x,y) = \frac{3x+4y}{x^2+y^2+1}$$

1. Produce a contour plot of $f(x,y)$ for $-2 \le x \le 2$ and $-2 \le y \le 2$. Are there any local maxima/minima in the rectangle? Where are they (approximately, from the plot)?

2. Produce a 3D plot of $f(x,y)$ for $-2 \le x \le 2$ and $-2 \le y \le 2$. Click on the plot and drag the mouse to see the plot rotate. Right-click on the plot and change the plotting style to see it in other ways. Discuss and compare to the contour plot. Is this function likely to have additional local maxima/minima outside the square $-2 \le x \le 2$ and $-2 \le y \le 2$? Why or why not?

3. Define the partial derivative functions f_x and f_y, and find critical points with Maple.

4. Define discriminant $DD(x,y)(= f_{xx}f_{yy} - f_{xy}^2)$ and $f_{xx}(x,y)$, and use them to classify the critical points.

A continous function $f(x,y)$ defined on a closed and bounded *region (see the text for definition of closed and bounded) is known to have a global (or absolute) maximum and minimum value there. These may occur inside or on its boundary. In Part 2 we will consider the fuction above but restricted to the disk $x^2+y^2 \le 4$.*

Part 2

Consider the function

$$f(x,y) = \frac{3x+4y}{x^2+y^2+1}$$

restricted to the disk $x^2+y^2 \le 4$.

(a) Using Maple and the Lagrange multiplier method, find the max and min values of $f(x,y)$ subject to the constraint $x^2+y^2 = 4$.

(b) Comparing these results with those in Part 1, find the global max and min of f on the disk $x^2+y^2 \le 4$.

Part 3

This time we restrict the function $f(x,y)$ to a different disk.
Find the global max and min of $f(x,y)$ on the disk $x^2+(y-2)^2 \le 4$.

USEFUL MAPLE COMMANDS

```
> restart;                   # good to have this at the top of worksheet.
> with(plots):               # extra functionality for plotting,
                             #     insert it near the top of worksheet.
> f:=x->x^2;                 # define a function f(x) of one variable.
> f:=(x,y)->2*x+3*y;         # define a function f(x,y), of two variables.
> evalf(%);                  # produce a decimal approximation to previous output.
> plot(f(x),x=-1..1);        # plot f(x) a function of one variable.
> plot(f(x),x=-1..1,y=0..2); #a plot where y-range is specified boxed.
> plot([f(x),g(x)],x=0..2);# plot two functions for x between 0 and 2.
> plot3d(f(x,y),x=0..2,y=0..3); # 3d plot of a 2 variable function.
> contourplot(f(x,y),x=0..2,y=0..3,grid=[15,15], color=black,contours=20);
                             # contour plot of 2 var. function with 20 contours.
> contourplot(f(x,y),x=0..2,y=0..3,grid=[15,15], contours=[1,2,3,4]);
                             # contour plot  with contours f=1, f=2,...,f=5
> solve(f(x)=g(x),x);        # solve the equation f(x) = g(x) for x.
> solve({F(x,y)=0,G(x,y)=0},{x,y}) # solve two eqns in two unknowns
> Pi ;                       # the constant 3.14...Note the it begins with capital P.
> exp(2.5);                  # exponential function evaluated at 2.5
> ln(2.5);                   # the natural logarithm of 2.5
> 2^(1/2)                    # the square root of 2; sqrt(2) is also valid.
> 2^(1/3)                    # the cubic root of 2
> fx:=D[1](f)     # define as a function the partial of f(x,y) with respect to x
> fxy:=D[1,2](f) # define as a fuction the 2nd partial of f with respect to x and y
> DD:=(x,y)->fxx(x,y)*fyy(x,y)-(fxy(x,y))^2  # the discriminant as a function
```