MTH 243 Calculus III Maple Homework #1

Graphs of Functions of Two Variables/Contour Diagrams. Due:

Note: Be sure to explain all your work clearly and in complete sentences.Type all comments/explanations in Maple in your homework sheet.

Problem1. A laterally insulated copper bar 70 cm long is located along the x axis with one end at x=0 and the other x=70. The ends of the bar are kept at constant temperature of 0^o C. Assume that the temperature $u(x, t)$ at any point x on the bar at time t, measured in minutes, is given by

$$u(x, t) = 100 \sin\left(\frac{\pi x}{70}\right) e^{(-0.18 t)}.$$

(a) Plot a few cross-sections with t fixed in one coordinate system, e.g., u(x,0), u(x,5), u(x,10), u(x,15). Explain their meaning in practical terms. In particular, what does u(x,0) tell you?
(b) Plot a few cross-sections with x fixed in one coordinate system, e.g., u(10,t), u(20,t), u(60,t), for t between 0 and 25. Explain their meaning in practical terms.
(c) Plot the 3d graph of the function u(x,t) showing the temperature throughout the bar during the first 25 minutes.
(d) By looking at the graph in (c), can you see at what point of the bar the temperature is always the highest? Can you explain this in physical terms? What happens to the temperature of the bar in the long run?

Problem 2. Consider the function

$$m(x, y) = \cos\left(\frac{x^2 + y^2}{5}\right)$$

(a) Draw the graph of the function in the patchcontour style. Choose a range for x and y not too large, so that the shape of the function is clearly visible and the graph is not too choppy, but large enough so that the global behavior of the function is evident. What symmetries does the graph have? Describe the shape and the behavior of the graph in your own words.
(b) Create a contour map and explain how it reflects the shape of the graph. Do not use filled=true option in this example.

There are two examples below to assist you

Example (using cross sections to visualize with 3D graph). Consider the function:

$$f(x, t) = t\, e^{(-t (5 - x))} \quad \text{for } 1 \le x, x \le 4,\ 0 \le t.$$

Define the function $f(x, t)$. The syntax is similar to the syntax in the case of one variable.

```
> f:=(x,t)->t*exp(-t*(5-x));
```

Plot cross-sections. We use the plotting syntax that we learned last year. First we plot cross sections for fixed x (Note that t varies). Plotting cross sections for t fixed (x varies in this case) is analagous. Make sure you have the appropriate ranges for each case.

```
> plot([f(1,t),f(1.5,t),f(2,t),f(3,t),f(4,t)],
         t=0..6,color=[red,blue,green,cyan,yellow], labels=["t","f(a,t)"])
```

```
;
```

Cross sections for fixed t:

```
> plot([f(x,1),f(x,2),f(x,3)],x=1..4,color=[red,blue,green],labels=["x","f(x,b)"]);
```

The 3 dimensional plot. We shall see now how all that information translates into properties of the graph of the function $C=f(x,t)$ of two variables. The syntax for plotting functions of two variables is similar to the syntax in the case of one variable. The basic command is **plot3d**. There are many options and attributes that can be added to a 3d plot. We shall learn them as we go along.

```
> plot3d(f(x,t),x=1..4,t=0..6,labels=["x","t","f(x,t)"],axes=normal);
```

Look carefully at the 3d graph above. Do you see the cross-sections with x fixed and with t fixed that we drew before as functions of one variable?

Here's the same plot using some different options:

```
> plot3d(f(x,t),x=1..4,t=0..6,labels=["x","t","f(x,t)"],axes=framed,grid=[40,40]);
```

To learn more about various options for 3d graphs, and there are plenty of them, use Maple's on-line help by typing **?plot3d,option**.

Example (using contour diagrams to visualize 3D graphs). Examine the behavior of the function $g(x,y)=x^3-3x+y^3-3y$ for x, y near $x=0, y=0$. Create a contour map for the function.

Define the function:

```
> g:=(x,y)->x^3-3*x+y^3-3*y;
```

Plot the function with the ranges x, y between -2 and 2.

```
> plot3d(g(x,y),x=-2..2,y=-2..2,axes=framed,orientation=[150,70]);
```

The contour plot. Maple can create a contour diagram of the function with a simple command **contourplot**. The command, as well as the command display that we use below, is contained in the **plots** package. Hence, we load the package first and then ask for a contour map.

```
> with(plots):
```

```
> contourplot(g(x,y),x=-2..2,y=-2..2);
```

The patchcontour style. We can obtain a much fancier contour map by using a few additional options. Here you will see the contours before they are projected onto the xy-plane, that is, the intersections of the graph of the function with equally-spaced horizontal planes. Try the following:

```
> plot3d(g(x,y),x=-2..2,y=-2..2,axes=framed,style=patchcontour,orientation=[150,70]);
```

Here's yet another type of contour map:

```
> contourplot(g(x,y),x=-2..2,y=-2..2,contours=[-3.5,-3,-2.5,-2,-1.5,-1,-0.5,0,0.5,1,1.5,2,2.5,3,3.5],filled=true,coloring=[white,red]);
```

Remark. If you want to obtain a contour map that allows you to read clearly the steepness of slopes and the shape of a surface, you should always take equally-spaced values for level curves, as we did in the above example. When we do this, closely-spaced contours on a contour map indicate steep increase or decrease in elevation, while contours far apart indicate mild slopes.

```
>
```