1. a. \( f'(x) = 6x^2 + 6x - 72 = 6(x^2 + x - 12) = 6(x + 4)(x - 3) \). Hence from \( f'(x) = 0 \) we have that the critical points are \( x = -4, x = 3 \).

b. \( f''(x) = 12x + 6 \); then,
   \( f''(-4) = -42 \) ⇒ local max at \( x = -4 \); \( f''(3) = 42 \) ⇒ local min at \( x = 3 \).

c. \( f''(x) = 12x + 6 \), hence the only possible inflection point is \( x = -\frac{1}{2} \).
   Check: \( f''(-1) = -6 < 0, f''(0) = 6 > 0 \). We conclude that \( x = -\frac{1}{2} \) is an inflection point since there is a sign change of \( f'' \) at \( x = -\frac{1}{2} \).

2. a. \( g'(x) = 12x^3 + 24x^2 - 12x - 24 = 12(x^3 + 2x^2 - x - 2) \). Now the equation \( g'(x) = 0 \) has the three solutions \( x = -2, x = -1 \) and \( x = 1 \). This can be seen from factoring \( g'(x) \), or from some calculator work. The critical points are \( x = -2, x = -1, x = 1 \).

b. \( g''(x) = 36x^2 + 48x - 12 \). Then,
   \( g''(-2) = 36 \) ⇒ local min at \( x = -2 \).
   \( g''(-1) = -24 \) ⇒ local max at \( x = -1 \).
   \( g''(1) = 72 \) ⇒ local min at \( x = 1 \).

c. \( g''(x) = 36x^2 + 48x - 12 \). By solving \( g''(x) = 0 \) we get the possible inflection points:
   \( x = \frac{1}{3}(-2 - \sqrt{7}) \approx -1.548 \), and \( x = \frac{1}{3}(-2 + \sqrt{7}) \approx 0.2152 \).
   Check: \( g''(-2) = 36 > 0, g''(0) = -12 < 0, g''(1) = 72 > 0 \). We conclude that both are inflection points.

3. a. \( h'(t) = -e^{-t^2} - te^{-t^2}(-2t) = (-1 + 2t^2)e^{-t^2} \). Critical points: \( t = -\frac{1}{\sqrt{2}}, t = \frac{1}{\sqrt{2}} \).

b. \( h''(t) = -t(4t^2 - 6)e^{-t^2} \). Then,
   \( h''(-\frac{1}{\sqrt{2}}) = -2\sqrt{2}e \) ⇒ local max at \( t = -\frac{1}{\sqrt{2}} \).
   \( h''(\frac{1}{\sqrt{2}}) = 2\sqrt{2}e \) ⇒ local min at \( t = \frac{1}{\sqrt{2}} \).

c. \( h'''(t) = -t(4t^2 - 6)e^{-t^2} \).
   Possible inflection pts: \( t = 0, t = -\sqrt{2} \approx -1.225, t = \sqrt{2} \approx 1.225 \).
   The following sign changes of \( h'' \) prove that the points we found are inflection points:
   \( h''(-2) = \frac{20}{e^4} > 0, h''(-1) = -\frac{2}{e} < 0, h''(1) = \frac{2}{e} > 0, h''(2) = -\frac{20}{e^4} < 0 \).

4. a. \( R'(t) = \frac{4(1+t)}{2+2t+t^2} \), hence the only critical point is \( t = -1 \).

b. The second derivative is \( R''(t) = \frac{-4t(2+4t+2t^2)}{(2+2t+t^2)^2} \). Then,
   \( R''(-1) = 4 \) ⇒ local min at \( t = -1 \).

c. From the equation \( R''(t) = 0 \) we have the possible inflection points: \( t = -2, t = 0 \).
   The following sign changes of \( h'' \) prove that the points we found are inflection points:
   \( f''(-3) = -\frac{12}{25} < 0, f''(-1) = 4 > 0, f''(1) = -\frac{12}{25} < 0 \).
5. Note: in our text the endpoints of the interval are considered critical points (no derivative there), and also as possible points of local max or min. The following table reflects this.

<table>
<thead>
<tr>
<th>Kind of point</th>
<th>Point or Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical point</td>
<td>E, G, A, I</td>
</tr>
<tr>
<td>Inflection point</td>
<td>D, F</td>
</tr>
<tr>
<td>Local min</td>
<td>G</td>
</tr>
<tr>
<td>Local max</td>
<td>E, I, A</td>
</tr>
<tr>
<td>Global min</td>
<td>G</td>
</tr>
<tr>
<td>Global max</td>
<td>A</td>
</tr>
</tbody>
</table>

6. \( f\left(\frac{1}{3}\right) = 1 \Rightarrow \frac{a}{3}e^{b/3} = 1 \) and \( f'(\frac{1}{3}) = 0 \Rightarrow a(1 + \frac{b}{3})e^{b/3} = 0 \), which implies that either \( a = 0 \) or \( b = -3 \). The \( a = 0 \) case gives the trivial solution \( f(x) = 0 \) for all \( x \), which contradicts \( f\left(\frac{1}{3}\right) = 1 \), so we discard this case. Then we must have \( b = -3 \), which combined with \( \frac{a}{3}e^{b/3} = 1 \) gives \( a = 3e \). That we have a local maximum at \( x = 1/3 \) is clear from the first derivative test, or from a plot of \( f(x) \).

7. \( x \)-intercepts: 0, \(-2a\). \( y \)-intercepts: 0. Critical points: \( x = -a \). \( f''(-a) = 2 > 0 \), hence \( f(x) \) has a local minimum at \( x = -a \). Since \( f(x) \) is a parabola, the local minimum is also a global minimum. The figure corresponds to \( a > 0 \). If \( a \) is increased, the nonzero \( x \)-intercept moves away (left) from the origin, and the minimum moves “down”.

![Figure for problem 7 when \( a > 0 \).](image)

8. (a) \( \{f(0), f(3), f(4)\} = \{0, -135, -112\} \Rightarrow x_{\text{min}} = 3, x_{\text{max}} = 0, \)
(b) \( \{f(-5), f(-4), f(3), f(4)\} = \{185, 208, -135, -112\} \Rightarrow x_{\text{min}} = 3, x_{\text{max}} = -4, \)
(c) \( \{f(-2), f(2)\} = \{140, -116\} \Rightarrow x_{\text{min}} = 2, x_{\text{max}} = -2, \)
(d) \( \{f(4), f(10)\} = \{-112, 1580\} \Rightarrow x_{\text{min}} = 4, x_{\text{max}} = 10. \)

9. (a) \( \{f(-2), f(-\frac{1}{\sqrt{2}}), f(0)\} \approx \{0.036, 0.4288, 0.\} \Rightarrow x_{\text{min}} = 0, x_{\text{max}} = -\frac{1}{\sqrt{2}}, \)
(b) \( \{f(-2), f(-\frac{1}{\sqrt{2}}), f(\frac{1}{\sqrt{2}}), f(2)\} \approx \{0.036, 0.4288, -0.4288, -0.036\} \)
\( \Rightarrow x_{\text{min}} = \frac{1}{\sqrt{2}}, x_{\text{max}} = -\frac{1}{\sqrt{2}}, \)
(c) \( \{f(-0.5), f(0.5)\} = \{0.3894, -0.3894\} \Rightarrow x_{\text{min}} = 0.5, x_{\text{max}} = -0.5, \)
(d) \( \{f(-3), f(-1)\} = \{0.00037, 0.3678\} \Rightarrow x_{\text{min}} = -3, x_{\text{max}} = -1, \)
10. Let us call $m$ and $M$ the best bounds. A plot of $f(x)$ suggests the answer $M = f(1) \approx 0.3678$ and $m = 0$.

We verify this as follows: $f'(x) = e^{-x} - xe^{-x} = (1 - x)e^{-x}$, hence $x = 1$ is the only critical point. Since $f'(0.5) \approx 0.303$ and $f'(2.0) \approx -0.1353$, we see that there is a local maximum at $x = 1$ (which is also a global maximum, since there is only one critical point). Therefore, $M = f(1) \approx 0.3678$. To get $m$, we investigate behavior of $f(x)$ near the endpoints of the interval. Observe that $f(0.5) \approx 0.3032$ and that $\lim_{x \to \infty} f(x) = 0$. Therefore, $m = 0$.

11. The function to be optimized is cost $C$. If the length and width of the rectangle are called $x$ and $y$, and if the side with fancy fence has length $x$, the total cost of the fence is

$$C = 7.5x + 5y + 5x + 5y = 12.5x + 10y$$

We must reduce $C$ to a single variable. For this we use the fact that the total area is 2880:

$$xy = 2880 \quad \Rightarrow \quad y = \frac{2880}{x}$$

Substituting this value of $y$ into $C$ we have the formula

$$C = 12.5x + \frac{28800}{x}, \quad \text{valid for} \quad 0 < x < \infty$$

We shall need

$$C'' = 12.5 - \frac{28800}{x^2} \quad \text{and} \quad C''' = \frac{57600}{x^3}$$

Solving for $x$ in $C' = 0$ gives $x = \pm 48$, but the negative answer is discarded. So $x = 48$ is the only critical point, and since $C'''(48) \approx 0.52$, we have a local minimum at $x = 48$. Since this is the only critical point, $C$ also has a global minimum there. Substituting $x = 48$ into $y = \frac{2880}{x}$, we get $y = 60$. Conclusion: the optimal dimensions of the rectangle are $x = 48$, $y = 60$, and the optimal cost of the fence is $C(48) = 1200$.

12. If the rectangle has sides $x$ and $y$, the function to be minimized is the diagonal $d = \sqrt{x^2 + y^2}$. To reduce $d$ to a single variable, use the fact $xy = A$, where $A$ is a given constant which we assume known. Since $y = A/x$, we have

$$d = \sqrt{x^2 + \left(\frac{A}{x}\right)^2} = \sqrt{x^2 + \frac{A^2}{x^2}}, \quad \text{where} \quad 0 < x < \infty$$

Now $d' = \frac{2x - \frac{2A^2}{x^3}}{2\sqrt{x^2 + \frac{A^2}{x^2}}}$, hence $d' = 0 \Rightarrow x = \pm \sqrt{A}$. The negative answer is discarded, and we are left with a single critical point, namely $x = \sqrt{A}$. Now

$$d''(x) = \frac{2 \left( A^4 + 3A^2x^4 \right)}{x^6 \left( x^2 + \frac{A^2}{x^2} \right)^{3/2}} \quad \Rightarrow \quad d''(\sqrt{A}) = \frac{2\sqrt{2}}{\sqrt{A}} > 0$$
so \( x = \sqrt{A} \) gives a local minimum of \( d \) which is also a global minimum. Hence the dimensions of the rectangle of area \( A \) that has the shortest diagonals are \( x = \sqrt{A} \) and \( y = A/x = \sqrt{A} \).

13. See page 205 of the text.

14. 
\[
\frac{d}{dx} \left( \frac{\sinh t}{\cosh t} \right) = \frac{\cosh t \cosh t - \sinh t \sinh t}{\cosh^2 t} = \frac{\cosh^2 t - \sinh^2 t}{\cosh^2 t}
\]

15. A: CFT, B: IVT, C: MVT, D: EVT

16. a) Upper estimate = Right Sum = 146; Lower estimate = Left Sum = 122;
   b) From \( \Delta t |v(b) - v(a)| = \text{difference between upper and lower estimates} \), and the requirement that the difference be less than 0.1, we have that \( \Delta t \approx \frac{0.1}{0.99} = 0.1010101 \approx 0.0008333 \).
   c) From \( \Delta t = \frac{b-a}{n} \), we have 0.0008333 > \( \Delta t = \frac{0.8-0.2}{n} \), hence \( n > \frac{0.8}{0.0008333} \approx 720 \).

17. a) Upper estimate = Left Sum = 111; Lower estimate = Right Sum = 0.57;
   b) From \( \Delta t |Q'(b) - Q'(a)| = \text{difference between upper and lower estimates} \) and from the requirement that the difference be less than 0.0001, we have that \( \Delta t = \frac{0.0001}{|Q'(b) - Q'(a)|} < \frac{0.0001}{0.19-0.01} \approx 0.0005556 \).
   c) From \( \Delta t = \frac{b-a}{n} \), we have 0.0005556 > \( \Delta t = \frac{15-3}{n} \), hence \( n > \frac{15-3}{0.0005556} \approx 21600 \).

18. (a) 6, (b) 12, (c) 3, (d) 12, (e) 6, (f) 3.

19. a) Left sum = -5.6, Right sum = 5.0, \( \int_{0}^{10} f(t)dt \approx \frac{1}{2}(Left + Right) = -0.3 \).
   b) The average value of \( f(t) \) on \( 0 \leq t \leq 10 \) is \( \frac{1}{10-0} \int_{0}^{10} f(t)dt \approx \frac{1}{10} \cdot (-0.3) = -0.03 \).

20. a) Left sum = \( \frac{4375}{27} \approx 162.037 \), Right sum = \( \frac{4375}{27} \approx 162.037 \).
   b) The answers are the same due to the symmetry of the graph of the function \( f(x) = -x^2 + 10x \) with respect to the vertical line \( x = 5 \), and to the fact that \( f(x) \) increases for \( x \leq 5 \), and that \( f(x) \) decreases for \( x \geq 5 \). The Left and Right sums are not necessarily equal to the integral, even though Left=Right. Indeed, a calculator computation (using \texttt{fnInt} on a Texas Instruments calculator, for example) gives that the integral \( \approx 166.66 \).

21. Left sum = \( \left( \sin(0) + \sin(\frac{25}{9}) + \sin(\frac{100}{9}) \right)^\frac{5}{3} \approx -1.06241 \)
   Right sum = \( \left( \sin(\frac{25}{9}) + \sin(\frac{100}{9}) + \sin(25) \right)^\frac{5}{3} \approx -1.28307 \)

22. Left sum = \( \left( e^{(-1)^2} + e^{(-0.8)^2} + e^{(-0.6)^2} + e^{(-0.4)^2} + e^{(-0.2)^2} \right) 0.2 \approx 0.681156 \)
   Right sum = \( \left( e^{(-0.8)^2} + e^{(-0.6)^2} + e^{(-0.4)^2} + e^{(-0.2)^2} + e^{(-0)^2} \right) 0.2 \approx 0.80758 \)

23. (a) The furthest the car gets ahead of the truck is at \( t \approx 1.5 \) hrs. The separation then is approximately “3 rectangles” = 30 miles. However, at the end of the 7 hours the truck is ahead by approximately “12 rectangles” = 120 miles !
   b) Yes, they do meet at approximately \( t = 3.5 \) hours.
c) At time $t = 6$ hrs the car comes to a stop, while the truck keeps going at $\approx 34$ mph. At time $t = 7$, the car is going in direction opposite to that of the truck, with velocity = -10 mph, while the truck moves at 45 mph.

24.

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<th>3</th>
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<th>5</th>
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<th>7</th>
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<tbody>
<tr>
<td>$f(x)$</td>
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<td>1</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

25.

26. (a) $\int_0^5 3f(t) + 2g(t) \, dt = 3 \int_0^5 f(t) \, dt + 2 \int_0^5 g(t) \, dt = 3 \cdot 23 + 2 \cdot 15 = 99$

(b) Cannot be done: we need the value of $\int_2^5 g(t) \, dt$ to answer (b).

(c) $\int_0^5 g(t) \, dt = \int_0^3 g(t) \, dt + \int_3^5 g(t) \, dt$, hence $15 = \int_0^3 g(t) \, dt + 22$, so $\int_0^3 g(t) \, dt = -7$.

(d) $\int_5^0 g(t) \, dt = - \int_0^5 g(t) \, dt = -15$.

27.

<table>
<thead>
<tr>
<th>$x$</th>
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<th>3</th>
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<tbody>
<tr>
<td>$f(x)$</td>
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<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

28.

Critical points: $x = 2.3$ and $x = 4$; Inflection points: $x = 1$, $x = 3$ and $x = 6$. 