Short Answers – Practice Problems for Test 2 - MTH 141 Fall 2003

PART 1

$$a'(x) = -6x - \frac{5}{x^6} - \frac{1}{3\sqrt{x}}$$

$$b'(t) = -7e^t + (\ln 2)2^t - \left(\ln \frac{1}{2}\right)\left(\frac{1}{2}\right)^t$$

$$c'(t) = \frac{5}{x} + 3\sin x + \frac{2}{\cos^2 x}$$

$$d'(t) = \frac{-10t + 9t^2 - 20t^4}{(1-4t^3)^2}$$

$$e'(x) = \frac{7\sin x}{\cos^2 x} - \sin x - x\cos x$$

$$f'(t) = \frac{5}{(5t-2)^2 + 1}$$

$$g'(x) = 2\sin(x)\cos(x) - 2\cos(2x)$$

$$\frac{dh}{du} = \frac{(\ln 2)2^u}{2^u + 3}$$

$$\frac{ih}{dx} = 0.7\ln(\ln x \ + 2)^{-0.3}\,\frac{1}{x}$$

$$\frac{dj}{dv} = \frac{1}{3}(ve^{2v+1})^{-2/3}(e^{2v+1} + 2ve^{2v+1})$$

$$\frac{dk}{d\theta} = \frac{1}{4\sqrt{3-\sqrt{5-\theta}}\sqrt{5-\theta}}$$

$$\frac{dl}{dt} = \frac{\sqrt{t+2} - \frac{3+t}{2\sqrt{t+2}}}{t+2} + \frac{-1}{2}\left(\frac{3+t}{t+2}\right)^{-1/2}\frac{1}{(t+2)^2}$$

$$\frac{dm}{dx} = \frac{3}{2\sqrt{\tan(3x)}\cos^2(3x)}$$

$$\frac{dn}{dt} = Ke^{Kt}(\cos(Rt) + \sin(St)) + e^{Kt}(-R\sin(Rt) + S\cos(St))$$

$$\frac{do}{d\phi} = \frac{-2a}{(1+a\phi)^2}$$

$$\frac{dp}{d\psi} = \frac{-\sin\psi\cos\psi}{\sqrt{A^2 - \sin^2\psi}}$$

$$\frac{dq}{dt} = (ACt^2 + 2At + B + BC)e^{Ct}$$

$$\frac{dr}{dx} = 0$$

$$\frac{ds}{dx} = e^{\sin x}(\cos^2 x - \sin x)$$

$$\frac{dt}{d\theta} = \frac{A\cos(A\theta)}{1 - \sin(A\theta)}$$

$$\frac{du}{dx} = \arcsin(\frac{x}{A}) + \frac{x}{A\sqrt{1-(\frac{x}{A})^2}}$$
$$\frac{dv}{dx} = \frac{-1}{3x^2} - 2^{-x} + x(\ln 2)2^{-x}$$
$$\frac{dw}{dx} = (\ln 3)3^{x \ln x}(\ln x + 1) + 3(x \ln x)^2(\ln x + 1)$$
$$\frac{dx}{d\theta} = 3(\ln\theta)^2\frac{1}{\theta} - \frac{3}{\theta}$$
$$\frac{dy}{d\mu} = -2A\sqrt[3]{1 - A\mu^2} + \frac{4A^2\mu^2}{3(1-A\mu^2)^{2/3}}$$
$$\frac{dz}{d\eta} = 2A\eta e^{B\eta^2}\left(1 + B\eta^2\right)$$

PART 2:

1. 18
2. -11
3. 77
4. 2
5. $\frac{46}{16}$
6. -3
7. $y' = \dfrac{xy}{1-x^2}$
8. $y' = \dfrac{-e^y}{xe^y - 1}$
9. $y' = \dfrac{\cos(ax)}{\sin(ay)}$
10. $y' = \dfrac{y(x^2+y)}{x^2(5y^2+x)}$
11. (a.) $1^2 + 2 = (1)(3)$
    (b.) $\frac{dy}{dx}\Big|_{(1,3)} = \frac{y}{2y-x}\Big|_{(1,3)} = -1$, so the tangent line is $y = -x + 4$.
    (c.) We want (x, y) so that $\frac{dy}{dx} = 0$. Thus $y = 0$, and after plugging into the equation of the curve we get that there are no such points.
    (d.) We want (x, y) so that the denominator in $\frac{dy}{dx}$ is zero, thus $2y - x = 0$, that is, $x = 2y$. Upon substitution into the eqution of the curve we get $y = \pm\sqrt{2}$, hence there are two such points: $(2\sqrt{2}, \sqrt{2})$ and $(-2\sqrt{2}, -\sqrt{2})$.

12. (a.) $x = -2 + 3\cos t,\ y = 5 + 3\sin t, \quad 0 \le t \le 2\pi.$
(b.) $x = -8 + 13t,\ y = 3 - t, \quad 0 \le t \le 1.$
(c.) $x = t,\ y = t^3 + t,\ -2 \le t \le 1.$
(d.) $x = -2 + 2t,\ y = -4t,\ -\infty < t < \infty.$

13. (a.) $\sqrt{(t-1)^6 + 4(t-1)^4}$ and $\sqrt{5}$
(b.) Down, since $y'(2) = -4$ is negative.
(c.) $-4/3$
(d.) Yes, since $\sqrt{(t-1)^6 + 4(t-1)^4} = 0 \;\Rightarrow\; t = 1$

14. (a.) $3/4$
(b.) $3/2$

15. $f(t) \approx e + -2e(t-0) \approx 2.71828 - 5.43656\,t$

16. $g(t) \approx 0 + 0(t-0) = 0$

17. $h(x) = \frac{1}{2} + 0(x - \frac{\pi}{4}) = \frac{1}{2}$

18. $4 + 5x$

19. 0.00214

20. -0.0000211

21. Type: "0/0". Ans: 4/3.

22. Does not exist. (Note numerator $\to \infty$ and denominator $\to 0$).

23. Type: "∞/∞". Ans: 1/3.

24. 0, by direct substitution of $x = 0$.

25. ∞, or, does not exist.

26. Type: "0/0". Ans: $-1/2$.

27. 0, by direct substitution.

28. 0 (note that the numerator approaches $\pi/2$ as $t \to \infty$).

29. Type: "∞/∞". Ans: 0.

30. $x^{0.2}$

31. x

32. $e^{0.0001x}$

33. $\frac{dy}{d\theta} = \frac{200}{\cos^2\theta}$

34. (a.) $\frac{dy}{dt} = \frac{-g}{K}(1 - e^{-Kt})$
(b.) $\frac{-g}{K} = \frac{-g}{K}(1 - e^{-Kt}) \;\Rightarrow\; 0 = e^{-Kt}$, which has no solutions. Hence the answer is NO.