Practice Problems for Test 1 - MTH 141 Fall 2003

Sections 1.1–1.7 and 2.1–2.7

NOTE: This is a selection of problems for you to test your skills. The idea is that you get a taste of the *kind* of problems that may appear on tests. Of course, some problems in the actual test may be completely different to the problems found here. Further, the questions in this document may not represent all the material discussed in class. To be better prepared for the test, make sure you review assignments, quizzes, class notes, read the book, etc.

1. (a) Given the function $f(x) = 2 + \frac{1}{(x+1)^2}$, determine domain and range.

   (b) Given the plot of a function shown in Figure 1, determine domain and range

2. Knowing that one of the functions $f(x)$, $g(x)$ and $h(x)$ shown in the table is linear $y = mx + b$, another is exponential $y = ca^x$, and another is a power function $y = kx^2$, say which is which and find a formula for each. Note: use rounding to 2 decimals.

x	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$	1.010	1.213	1.457	1.750	2.102	2.525
$g(x)$	1.019	1.065	1.153	1.300	1.518	1.823
$h(x)$	1.639	1.722	1.805	1.888	1.971	2.054

3. Figure 2 shows the plot of an exponential function $y = f(t)$. Find its formula.

4. A tank is filled with a solution of water and salt. Fresh water is added continuously to the tank, and there is runoff so that the volume of the mix stays constant. Knowing that the total amount of salt Q (pounds) in the tank at time t (days) is decaying exponentially with continuous decay rate of 5% , determine how long it takes for the amount of salt in the tank to be reduced to 10% of the original amount.

5. Solve for x: (a) $3\ 2^x = 5$, (b) $7 = e^{2x-1}$

   Simplify: (c) $e^{5\ln x}$, d) $\ln(3e^{2x})$

6. (a) Plot the function $f(x) = a\sin(3x) + 3a$, knowing that a is a positive number. Label tickmarks on the x and y axes (in terms of the parameter "a" if necessary) so that the period and amplitud can be read from the plot.

   (b) Find a reasonable formula and period for the sinusoidal function shown in Fig. 3.

7. Knowing that the temperature T at Death Valley, Calif. oscillates between 140 degrees F at 3 p.m. and 75 degrees 3 a.m. during the first week of July, find a reasonable formula for T (degrees) as a function of time t (hours since 3 p.m.). You may assume that the function has sinusoidal form.

8. (a) Find a formula for a polynomial of the smallest degree that has the graph in Figure 4.

   (b) If in question (a) we also assume that the y intercept is 10, what is the formula?

9. Figures 5 and 6 show the plots of $y = x^3$ and $y = (2.5)^x$. Use your calculator to

   (a) find a suitable domain and range for the plot in Figure 5.

   (b) find a suitable domain and range for the plot in Figure 6.

   (c) find all solutions to the equation $x^3 = (2.5)^x$ (use parts (a) and (b)).

10. (a) is $f(x) = \frac{x+2}{1-x}$ continuous on the interval $-2 < x < 2$? Why or why not?

    (b) Use your calculator to plot the function $g(x) = \sqrt{(x-1)^2}$ on $-2 \leq x \leq 2$. Is $g(x)$ continuous at $x = 1$? Is $g(x)$ differentiable at $x = 1$? Why or why not?

11. For the function shown in Figure 7,

    a) Find all intervals where $f'(x) > 0$.

    a) Find all intervals where $f'(x) < 0$.

    c) For what value of x is $f'(x)$ the least?

12. For the function shown in Figure 8, find the value of the limits, or write "DNE".

$$(a) \lim_{x\to -1^-} f(x) = \qquad (b) \lim_{x\to -1^+} f(x) = \qquad (c) \lim_{x\to -1} f(x) =$$

$$(d) \lim_{x\to 1^-} f(x) = \qquad (e) \lim_{x\to 1^+} f(x) = \qquad (f) \lim_{x\to 1} f(x) =$$

13. Investigate $\lim_{t\to 0} (1+t)^{1/t}$ numerically by producing a table of values.

14. Investigate numerically $f'(2)$ for $f(x) = (0.35)^x$.

15. (a) Compute $f'(2)$ with the definition of derivative (as a limit) for $f(x) = 5x^2 - 4x + 1$. (b) Find the equation of the tangent line to $f(x)$ at $x = 2$.

16. For the function $f(x)$ shown in Figure 9, plot $f'(x)$.

17. Compute $f'(x)$ with the definition of derivative (as a limit) if $f(x) = 3 - 7x^2$.

18. (a) Use the table to compute approximately $f'(1.9)$. (b) Compute the average rate of change of f between $t = 1.8$ and $t = 2.1$. (c) Compute approximately $f''(1.9)$.

t	1.7	1.8	1.9	2.0	2.1	
$f(t)$	0.991	0.973	0.946	0.909	0.863	
$f'(t)$						

19. At price p dollars, the number of items sold at a store is $q = f(p)$. Explain in regular English the following statements: (a) $f(35) = 250$, and (b) $f'(35) = -23$. What are the units of $f'(x)$?

20. Draw the graph for a function $f(x)$ that satisfies the following: $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $x > 2$, $f''(x) > 0$ for $x < -1$ and $f''(x) < 0$ for $x > -1$.

21. For the function given in Figure 9,

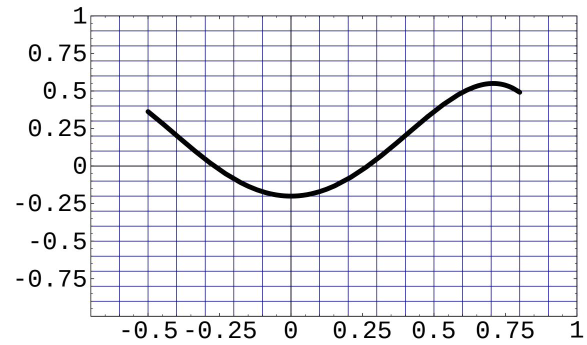
    a) Specify intervals where $f'' > 0$ and where $f'' < 0$.

    b) For what value of x is $f''(x)$ the greatest?

    c) For what value of x is $f''(x) = 0$?

22. For the function and the points shown in Figure 10, answer YES or NO in each slot.

POINT x	1	1.5	2.0	2.5	3.0	3.5
f is continuous at x						
f is differentiable at x						



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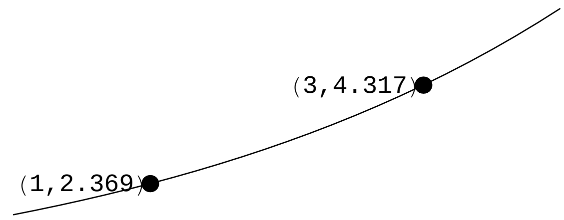

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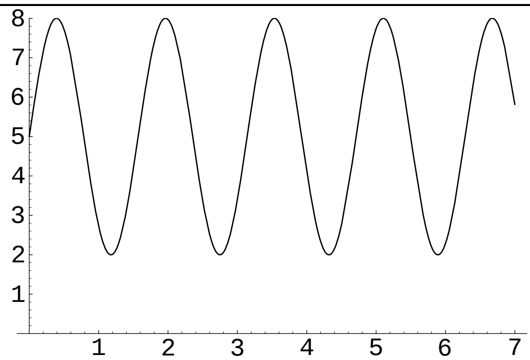

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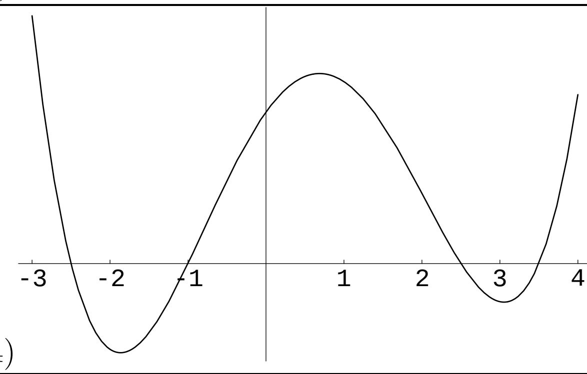

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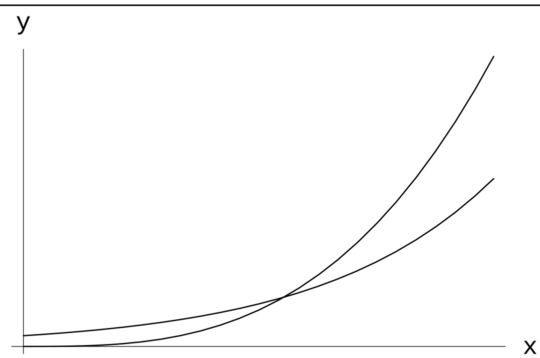

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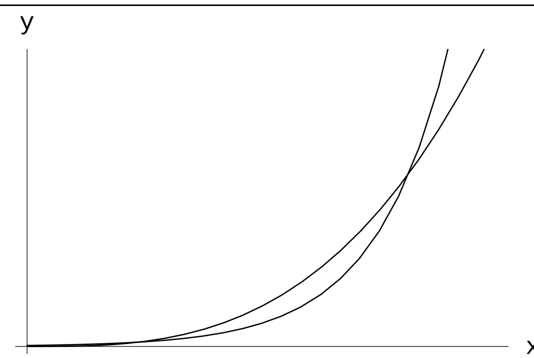

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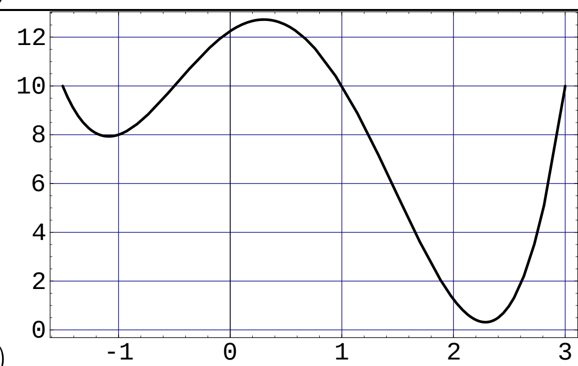

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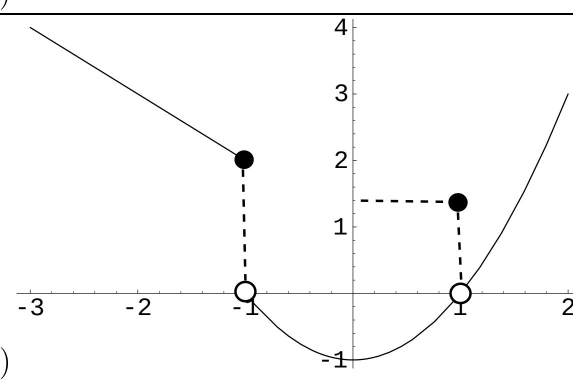

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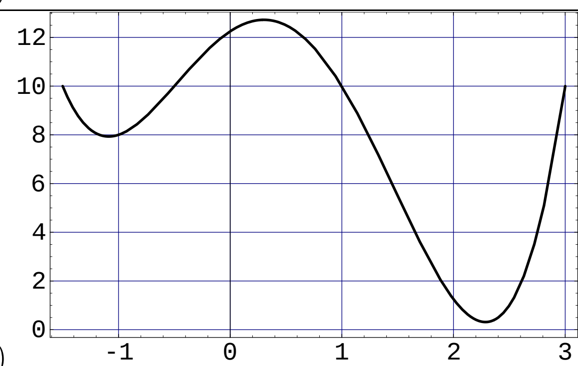

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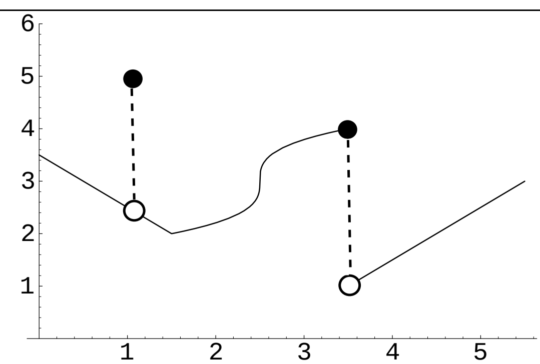

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