Maple Project 3 MTH 141 Fall 2003

The purpose of this homework is to use Maple to better understand Riemann sums and integration.

Consider the function

$$f(x) = 1.5 + \frac{x \sin x}{4}, \qquad 0 \le x \le \pi$$

Use Maple to answer the following questions. See page 2 for instructions and Maple hints.

PART 1.

1.1 Produce a plot of *left endpoint rectangles* for $0 \le x \le 4$ and a plot of *right endpoint rectangles*. Use $n = 9$ subintervals.

1.2 Have Maple calculate a *Left-Sum* of $f(x)$ and *Right-Sum* of $f(x)$ on $0 \le x \le \pi$ with $n = 9$ subintervals, first in symbolic (Σ) form, then as a decimal approximation.

1.3 By looking at the plots say whether the left Riemann sum *Left(n)* for general n gives a lower estimate, upper estimate or neither to $\int_0^\pi f(x)\,dx$, and explain why.

PART 2

2.1 Compute a decimal approximation to $A = \int_0^\pi f(x)\,dx$ by using Maple's `int` command.

2.2 Define the error as a Maple function of n as follows, where $Left(n)$ means the left Riemann sum of $f(x)$ with n subdivisions on $0 \le x \le \pi$:

$$Error(n) = A - Left(n)$$

Note that n represents an integer. (hint: $Error(1) = 0.785398163$. Use this information to check your $Error(n)$ function.)

2.3 Create a list of pairs [[1,Error(1)],... ,[20,Error[20]]], and use it to produce a plot of the Error (vertical axis) vs. n (horizontal axis), for $0 \le n \le 20$, $n \in N$. Specify a vertical range $-0.1 \le y \le 0.1$. Comment on whether the plot suggests that the left sum gives an overestimate or an underestimate for most n values, and say why.

2.4 Experiment with various n values to get the smallest n for which the error falls between -0.001 and 0.001.

INSTRUCTIONS and additional information

- Maple homework should have only one author. You may discuss the project with your classmates, but what you turn in should contain your own answers. Plagiarism is a serious offense.
- Write your name, class and section at the top of the worksheet.
- Precede Maple commands with a comment on what is being done. Neatness and good English will be taken into account.
- Maple should be used in all calculations and plots. The first Maple input line should be
  `restart; with(plots): with(student):`
  This loads special functions needed for this project.
- MAPLE HELP is available in Tyler 101, see `www.math.uri.edu/Courses/fall03/mth141`

SOME USEFUL MAPLE COMMANDS

```
> restart;                # good to have this at the top of worksheet;
> with(student):          # add this at the top of your worksheet.
                          # in particular, defines commands like leftbox and leftsum.
> with(plots):            # adds functionality for plotting.
> f:= x -> x^2;           # define a function f(x)
> D(f)(x);                # derivative of f at x
> plot(f(x),x=-1..1,y=0..2); # plot f(x)
> fsolve(f(x)=0,x=2..3); # find a solution to f(x)=0 between 2 and 3.
> Pi;                     # the constant 3.1415...Note the it begins with capital P.
> leftbox(f(x),x=a..b);  # produces a plot of left Riemann sum rectangles.
> rightbox(f(x),x=a..b); # produces a plot of right Riemann sum rectangles.
> leftsum(f(x),x=a..b,n);# left Riemann sum of f(x) with n subintervals on a<x<b.
> rightsum(f(x),x=a..b,n);# right Riemann sum of f(x) with n subintervals on a<x<b.
> evalf(%);               # evaluate previous output in decimal form.
> int(f(x),x=a..b);       # integral of f(x) from a to b.
> mypoints:=evalf([seq([n,g(n)],n=1..10)]); # a list of pairs, named  ‘‘mypoints’’.
> plot(mypoints,n=0..10,y=0..3);            # plot ‘‘mypoints’’ on the given window.
```