Chapter 7 Linear Algebra

Matrices, Vectors, Determinants,
Linear Systems.

§7.1 Matrices, Vectors: Addition and Scalar Multiplication

Matrix – a rectangular (2D) array of numbers, called the entries of the matrix.

Examples

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$C = \begin{bmatrix} e^{-x} & 2x^2 \\ 6^x & i \end{bmatrix}, \quad V = [v_1, v_2, v_3], \quad W = \begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix}$$

A has two <u>rows</u> (horiz. lines of entries)
and two <u>columns</u> (vert. lines of entries)

In general, if a matrix M has m rows and n columns, we say M is $m \times n$.

So here A is 2×3.

B, C are examples of square matrices which have the same number of rows as columns (B is 3×3, C is 2×2).

B is an example of a general matrix, the exact values of whose entries are unspecified.
Here b_{ij} refers to the (unique) entry in row i and column j (like an address).
e.g. b_{23} - entry in row 2 & column 3.

We often write general matrices as

$$M = (m_{ij})_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}}$$

(here M is $m \times n$).

or, more explicitly

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & & \\ \vdots & & & \\ m_{m1} & \cdots & & m_{mn} \end{bmatrix}$$

If M is $n \times n$ square, the entries $m_{11}, m_{22}, \ldots, m_{nn}$ are called the <u>diagonal entries</u> of M.

v, w are examples of <u>vectors</u> which are matrices with just a single row or a single column.

Here v is a <u>row vector</u> (1×3)
w is a <u>column</u> vector (2×1)

General Row Vector

$$v = [v_1, \ldots, v_n]$$

$1 \times n$

General Column Vector

$$w = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix} \quad m \times 1.$$

Equality of Matrices

Two matrices $A = (a_{ij})$, $B = (b_{ij})$ of the same size $(m \times n)$ are equal if all the corresponding entries are equal

ie. $a_{ij} = b_{ij}$ for each $1 \leq i \leq m$, $1 \leq j \leq n$

Write $A = B$.

Ex. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

Then $A = B$ means precisely that

$a_{11} = 1$, $a_{12} = 2$, $a_{21} = 0$, $a_{22} = 1$.

Addition of Matrices

If $A = (a_{ij})$, $B = (b_{ij})$ are two matrices of the same size $(m \times n)$, then the sum $A + B$ is the $m \times n$ matrix which has entries $a_{ij} + b_{ij}$ (just add the entries in corresponding positions).

Ex. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -6 & 1 \\ 0 & 1 \\ 2 & 7 \end{bmatrix}$

$$A + B = \begin{bmatrix} -5 & 3 \\ 3 & 5 \\ 3 & 9 \end{bmatrix}$$

Scalar Multiplication

If $A = (a_{ij})$ is $m \times n$ and c is a number, then the scalar product of A by c is the matrix $cA := (ca_{ij})$ (just multiply every entry in A by c).

Ex.

$$A = \begin{bmatrix} i \\ 3 \\ 6 \end{bmatrix}, \quad c = i$$

$$cA = \begin{bmatrix} -1 \\ 3i \\ 6i \end{bmatrix}.$$

Rules for Matrix Addition and Scalar Multiplication

Let A, B, C be $m \times n$ matrices. Then

a) $A + B = B + A$

b) $(A+B)+C = A + (B+C)$ (written $A+B+C$).

c) $A + O = A$

d $A + (-A) = O$.

Here $O = (0)_{\substack{1 \le i \le m \\ 1 \le j \le n}}$ is the $m \times n$ zero matrix with zeroes in all its entries while if $A = (a_{ij})$, then $-A = (-a_{ij})$ is obtained from A by multiplying by the scalar -1.

If in addition, c, k are scalars, then

e) $c(A+B) = cA + cB$

f) $(c+k)A = cA + kA$

g) $c(kA) = (ck)A$ (written ckA)

h) $1A = A$.