MTH 362: Fall 2005
Review for Test II
Test II will cover 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.1.

1. Verify that y is a solution of the differential equation. Determine c so that the resulting particular solution satisfies the given initial condition.
   (a) $x^3 + y^3y' = 0, x^4 + y^4 = c(y > 0), y(0) = 1.$
   (b) $xy' = 3y, y = cx^3, y(-4) = 16.$

2. Solve the following equations and the initial value problems by separation of variables
   (a) $y' + 3x^2y = 0$
   (b) $e^xy' = 2(x+1)y^2, \quad y(0) = \frac{1}{6}$
   (c) $xy' = y^2 + y$ (use $y/x = u$)

3. Experiments show that the rate of inversion of cane sugar in dilute solution is proportional to the concentration $y(t)$ of unaltered sugar. Let the concentration be $1/100$ at $t = 0$ and $1/300$ at $t = 4$ hours. Find $y(t)$.

4. An airplane taking off from a landing field has a run of 2 kilometers. If the plane starts with a speed of 10 m/s, moves with acceleration 1.5 m/s^2, with what speed does it take off?

5. Solve the following linear differential equations and the Bernoulli equation.
   (a) $y' + 2xy = 4x$
   (b) $y' + 3y = \sin x, \quad y(\frac{\pi}{2}) = 0.3$
   (c) $y' + xy = xy^{-1}$

6. Verify which equations are exact. For those that are exact find its general solution.
   (a) $(4x^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$
   (b) $(x^2 + y^2 + x)dx + xydy = 0$
   (c) $(2x^3 + 3y)dx + (3x + y - 1)dy = 0.$

7. (a) Use Kirchhoff's law to write the initial value problem – ODE and initial condition – for the simple circuit consisting of a 60 volt DC battery connected in series with a 4 henry inductor and a 12 ohm resistor. Current flows when the open switch is closed.
   (b) Verify that $I(t) = 5(1 - e^{-3t}), t \geq 0$ is the solution to the IVP in (a).
   (c) Graph $I(t)$. What is the asymptotic limit of $I(t)$ as $t \to \infty$. This is called the steady state current and will be denoted by I_∞.
   (d) At what time t does the current $I(t)$ reach 99% of its steady state value?

8. Verify that the given functions y_1 and y_2 form a basis of solutions of the given equation and solve the given initial value problem.
$$4x^2y'' - 3y = 0, y(1) = 3, y'(1) = 2.5; y_1 = x^{-1/2}, y_2 = x^{3/2}$$

9. Show that $y_1(t) = \sqrt{t}$ and $y_2(t) = \frac{1}{t}$ are solutions of the differential equation
$$2t^2y'' + 3ty' - y = 0.$$
Show that the $y_1(t)$ and $y_2(t)$ are linearly independent solutions for the ODE.

10. Show that $y_1 = x^3$ is a solution of the equation

$$x^2y'' + xy' + 9y = 0$$

Then use reduction of order to find y_2.