MTH 362: Fall 2005
Answers to Review II

1. (a) By implicit differentiation: $y' = -\dfrac{x^3}{y^3}$ plugging into the ODE

$$x^3 + y^3\left(-\frac{x^3}{y^3}\right) = x^3 - x^3 = 0$$

$c = 1.$

(b) $y' = 3cx^2$ plugging into the ODE

$$x(3cx^2) = 3cx^3 = 3y$$

$c = -1/4.$

2. (a) $y = Ce^{-x^3/3}$

(b) $y = \dfrac{e^x}{2x + 4 + 2e^x}$

(c) $y = -\dfrac{x}{x + C}$

3. $y(t) = e^{-(\ln 3/4)t}$

4. $s(t) = 0.75t^2 + 10t$, $v_{final} = v(45.4) = 78.1m/s = 281km/hr.$

5. (a) $y = 2 + Ce^{x^2}$

(b) $y = \dfrac{1}{10}(3\sin x - \cos x)$

(c) $y^2 = 1 + Ce^{x^2}$

6. (a) $\dfrac{\partial M}{\partial y} = 12x^3y^2 - 2x = \dfrac{\partial N}{\partial x}$ so the equation is exact, its solution is given by $x^4y^3 - x^2y = C.$

(b) $\dfrac{\partial m}{\partial y} = 2y, \dfrac{\partial N}{\partial x} = y$; so the equation is not exact.

(c) $\dfrac{\partial M}{\partial y} = 3 = \dfrac{\partial N}{\partial x}$ and the equation is exact. Solution is $x^4 + 6xy + y^2 - 2y = C.$

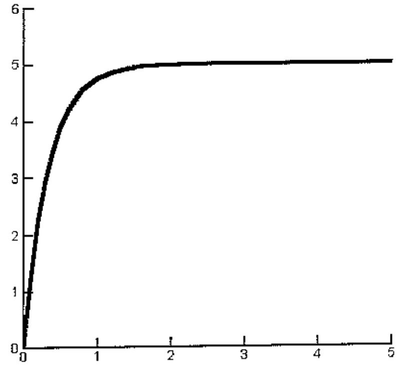
7. (a) By Kirchhoff's Law: $E_L + E_R = E$

$$\begin{aligned} 4\frac{dI}{dt} + 12I &= 60 \\ I(0) &= 0 \end{aligned}$$

(b) $I'(t) = 15e^{-3t}$, plugging into the ODE

$$4(15e^{-3t}) + 12(5(1 - e^{-3t})) = 60e^{-3t} + 60 - 60e^{-3t} = 60$$

(c) $I_\infty = 5$



(d) $t = 1.535$.

8.

$$y_1'(t) = \tfrac{1}{2}t^{-1/2} \qquad y_2'(t) = -\tfrac{1}{t^2}$$
$$y_1''(t) = -\tfrac{1}{4}t^{-3/2} \quad y_2''(t) = \tfrac{2}{t^3}$$

Substituting y_1 and its derivatives into the ODE, we have

$$2t^2 \cdot (-\frac{1}{4})t^{-3/2} + 3t(\frac{1}{2}t^{-1/2}) - t^{1/2} = -\frac{1}{2}\sqrt{t} + \frac{3}{2}\sqrt{t} - \sqrt{t} = 0.$$

Substituting y_2 and its derivatives into the ODE, we have

$$2t^2(\frac{2}{t^3}) + 3t(-\frac{1}{t^2}) - \frac{1}{t} = \frac{4}{t} - \frac{3}{t} - \frac{1}{t} = 0.$$

To determine if the solutions are linearly independent

$$\frac{y_1}{y_2} = \frac{\sqrt{t}}{\frac{1}{t}} = t\sqrt{t} \neq \text{ constant}$$

9.

$$y_1'(t) = 3x^2$$
$$y_1''(t) = 6x$$

Plugging into the ODE

$$x^2(6x) + x(3x^2) - 9(x^3) = 6x^3 + 3x^2 - 9x^2 = 0$$

$y_2 = -\dfrac{1}{6x^3}$.